$$\pi_0(X) = \text{set of connected components of } X$$

is a homotopy invariant: $X \simeq Y \Rightarrow \pi_0(X) = \pi_0(Y)$.

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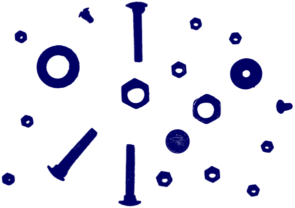
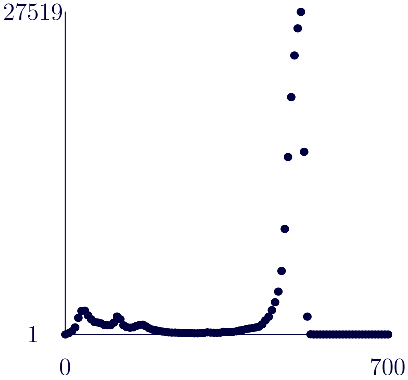
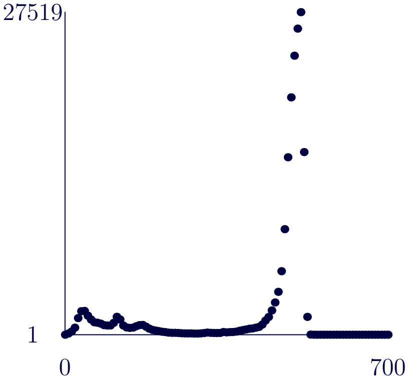
$$S_t = \{(x, y) \in \mathbb{R}^2 \ : \ ||f(x, y)|| \leq t\}$$





Plot of $|\pi_0(S_t)|$ as a function of t



Plot of $|\pi_0(S_t)|$ as a function of t

$t_1 < t_2 < \cdots < t_T$ implies $S_{t_1} \subset S_{t_2} \subset \cdots \subset S_{t_T}$

$$\beta_0^{t,t'} = |\mathsf{image}(\ \pi_0(S_t) \to \pi_0(S_{t'})\ )| \text{ for } t \leq t'$$

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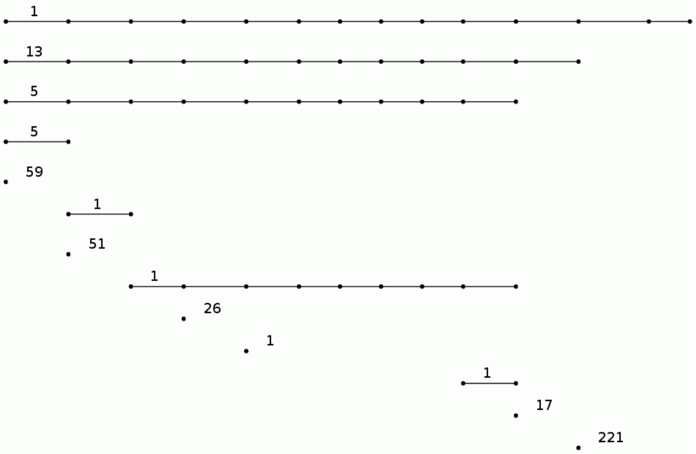

r lines from column t to column t' if $\beta_0^{t,t'} = r$

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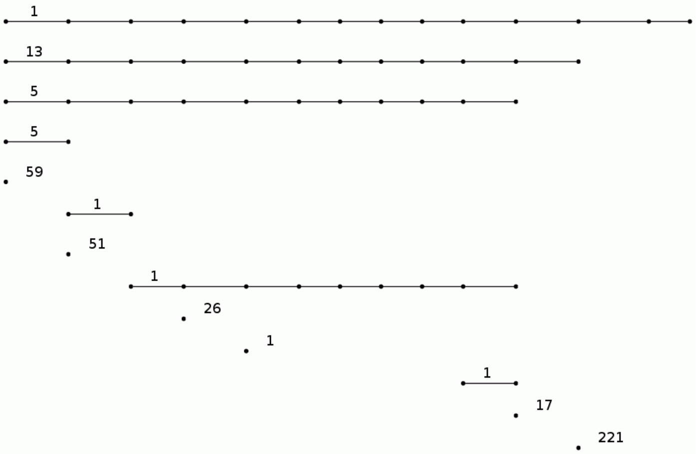

r lines from column t to column t' if $\beta_0^{t,t'} = r$

There are 20 objects in the photo.

To see how many objects have holes in them, consider

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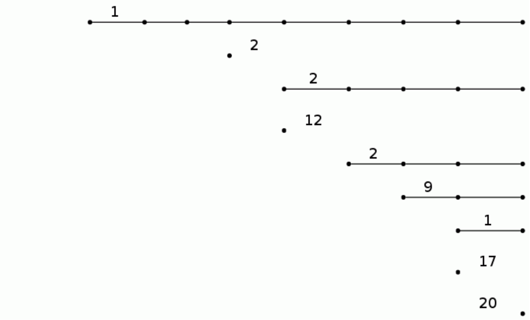

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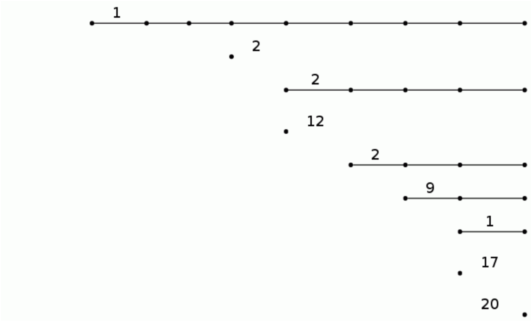

The photo has 14 objects with holes.