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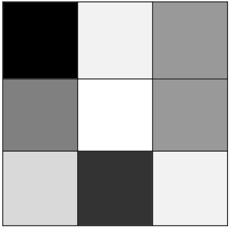

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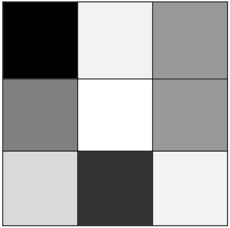

By normalizing with respect to brightness and contrast the patches are projected onto a set of points $\mathcal{M}$ in a topological seven-sphere $S^7 \subset \mathbb{E}^8$.

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$\mathcal{M}(k)$ *contains the* 25% *of patches with least* $\delta_k(x)$.

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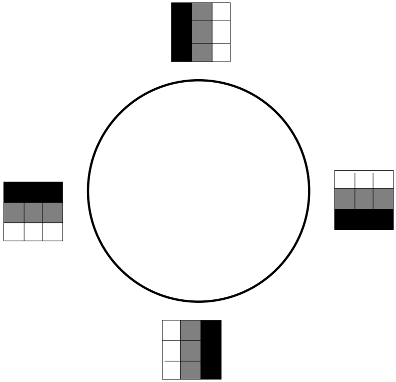

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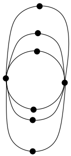

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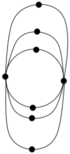

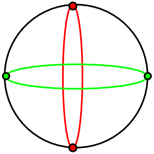

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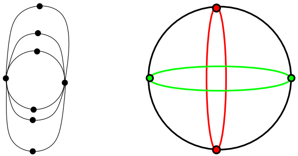

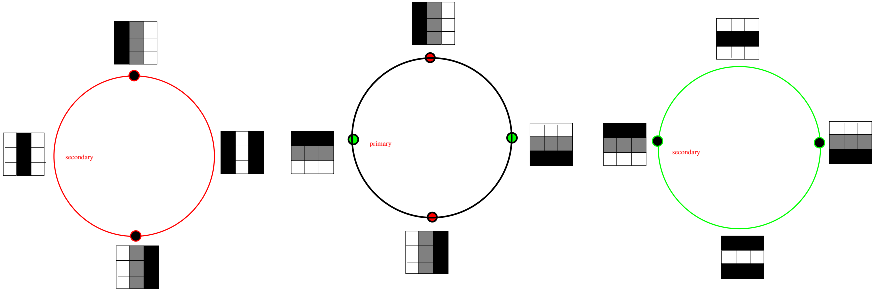

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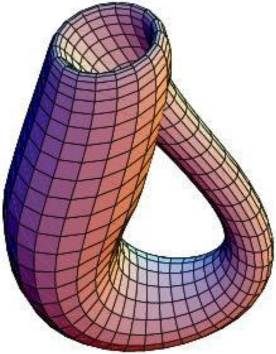

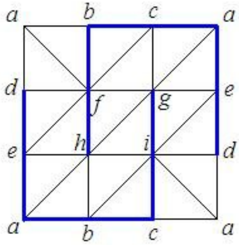

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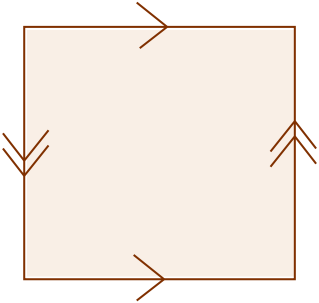

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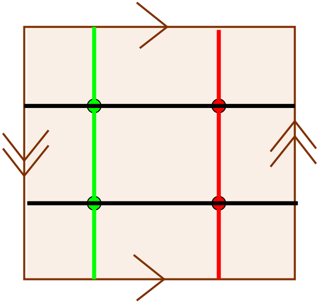

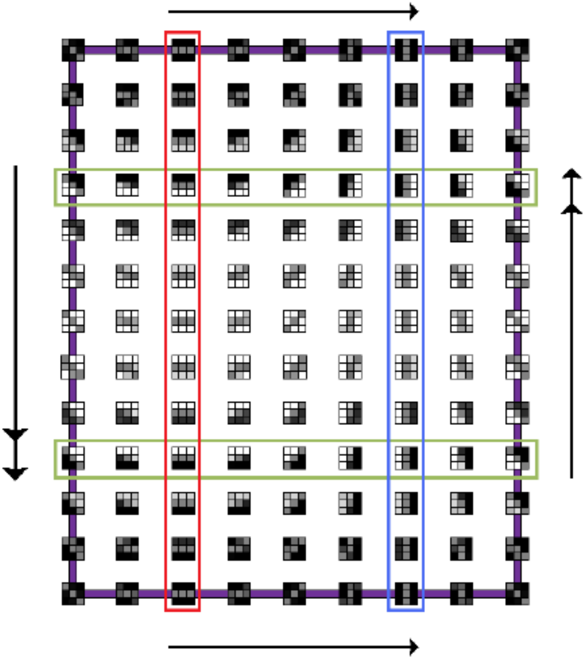

Theorem. [Edelsbrunner, Chazal, ...] For two finite metric spaces S, S' and $n \geq 0$ we have

$$d_{BottleNeck}(\beta_n^{**}(S), \beta_n^{**}(S')) \leq d_{GromovHaussdorf}(S, S') \ .$$

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Define the **Gromov-Hausdorff distance** *between* X *and* Y *to be*

$$d_{GH}(X, Y) = \frac{1}{2} \inf_{C \in \Gamma(X,Y)} \sup_{(x,y),(x',y') \in C} |d_X(x, x') - d_Y(y, y')|.$$

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Define a δ-**matching** *between barcodes* C *and* D *to be a bijection* $C^{2\delta} \leftrightarrow D^{2\delta}$ *such that: the bijection associates each interval in* $I \in C^{2\delta}$ *with an interval in* $\sigma(I) \in D^{2\delta}$*; if* $\sigma(I) = J$ *then* $J \subset \tau_\delta(I)$ *and* $I \subset \tau_\delta(J)$.

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Now define the **bottleneck distance** *between two barcodes* C, D *to be*

$$d_B(C, D) = \inf\{\delta : \text{there is a } \delta\text{−matching between } C \text{ and } D\}.$$