



## Semester 1 Examinations 2018/2019

<b>Exam Codes</b>	2BA1, 3BO2, 2BS1, 2BPT1, 2BCS1.
<b>Exams</b>	Second Year Arts and Science.
<b>Module Code</b>	MA 211.
<b>Modules</b>	Calculus I.
<b>Paper No</b>	1.
<b>Repeat Paper</b>	No.
<b>External Examiner</b>	Prof. T. Brady,
<b>Internal Examiners</b>	Prof. G. Ellis, Dr M. Makrooni.
<b>Instructions</b>	Answer any <b>THREE</b> of the four questions.
<b>Duration</b>	<b>TWO HOURS.</b>
<b>No. of Pages</b>	3 pages.
<b>Requirements:</b>	
Release in Exam venue	Yes <input checked="" type="checkbox"/> No <input type="checkbox"/>
Release to Library	Yes <input checked="" type="checkbox"/> No <input type="checkbox"/>
MCQ	Yes <input type="checkbox"/> No <input checked="" type="checkbox"/>
Statistical / Log Tables	Yes <input checked="" type="checkbox"/> No <input type="checkbox"/>
Other Materials	Non-programmable calculators are permitted

1. (a) Explain how to add two vectors geometrically and how to add them algebraically. [2 marks]
- (b) If  $\mathbf{a} = \langle 1, -3, 7 \rangle$  and  $\mathbf{b} = \langle 2, -1, -5 \rangle$ , find
- (i)  $\mathbf{a} - 3\mathbf{b}$ . [2 marks]
- (ii)  $|\mathbf{a} + \mathbf{b}|$ . [2 marks]
- (iii) the unit vector that has the same direction as  $\mathbf{a}$ . [2 marks]
- (c) Use the scalar triple product to show that the vectors  $\mathbf{a} = \langle 1, 4, -7 \rangle$ ,  $\mathbf{b} = \langle 2, -1, 4 \rangle$  and  $\mathbf{c} = \langle 0, -9, 18 \rangle$  coplanar. [6 marks]
- (d) Find the angle between the planes  $x + y + z = 1$  and  $x - 2y + 3z = 1$ . [6 marks]
2. (a) Let  $\mathbf{r}(t) = t \cos(t)\mathbf{i} + t^2\mathbf{j} + e^t\mathbf{k}$ .
- (i) Find  $\lim_{t \rightarrow 0} \mathbf{r}(t)$ . [2 marks]
- (ii) Find  $\mathbf{r}'(t)$ . [3 marks]
- (ii) Find  $\int \mathbf{r}(t) dt$ . [3 marks]
- (b) The position vector of an object moving in a plane is given by  $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j}$ . Find its velocity, speed and acceleration when  $t = 1$ . [6 marks]
- (c) Show that the curvature of a circle of radius  $a$  is  $1/a$ . [6 marks]

3. (a) Let  $f(x, y) = \sqrt{9 - x^2 - y^2}$ .
- (i) Find the domain and the range of  $f$ . [6 marks]
- (ii) Sketch the graph of  $f$ . [2 marks]
- (b) Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$  if it exists or show that this limit does not exist. [6 marks]
- (c) Let  $f(x, y) = 16 - 4x^2 - y^2$ . Find  $\frac{\partial f}{\partial x}(1, 2)$  and  $\frac{\partial f}{\partial y}(1, 2)$ . [6 marks]

4. (a) Find the equation of the tangent plane to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$

- at the point  $(-2, 1, -3)$ . [6 marks]
- (b) Find the directional derivative of the function  $f(x, y) = x^2y^3 - 4y$  at the point  $(2, -1)$  in the direction of the vector  $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$ . [6 marks]
- (c) Find the shortest distance from the point  $(1, 0, -2)$  to the plane  $x + 2y + z = 4$ . [8 marks]