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★**Persistence theory: from quiver representations to data analysis.**
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The concept of *persistence* has its main roots in topology and starts from the study of the homology of the sublevel sets $f^{-1}((-\infty, t])$ of a continuous real-valued function f [H. Edelsbrunner and D. Morozov, in *European Congress of Mathematics*, 31–50, Eur. Math. Soc., Zürich, 2013; MR3469114]. In plain words, a homology class is "persistent" if it "persists" with respect to changes of the value t defining the sublevel set.

This book mainly concerns the algebraic aspects of persistence theory, focusing on persistence modules, zigzag modules and related concepts. The author bases his exposition on the concepts of *quiver* and *quiver representation*, illustrating an interesting unifying setting. Put simply, a quiver can be seen as a directed multigraph, possibly endowed with infinitely many nodes and arrows, while a representation of a quiver Q over a field k is a "realization" of Q as a (possibly non-commutative) diagram of k-vector spaces and k-linear maps. This book shows how the language of quivers can be fruitfully used as a general approach to the algebraic aspects of persistence theory.

The book consists of three parts. In the first part (Chapters 1–3) algebraic persistence, topological persistence and their stability are presented. It focuses on the Isometry Theorem, which states the equality of the *bottleneck distance* between the persistence diagrams of two $\mathfrak{q}$-tame persistence modules over $\mathbb{R}$ and their *interleaving distance*. The second and longest part (Chapters 4–7) is devoted to applications, focusing on topological inference, clustering and data comparison through signatures obtained by persistence theory. This part presents an interesting survey of methods to efficiently manage the computation of persistence via the choice of suitable simplicial structures. The last part (Chapters 8–9) lists some open questions and possible lines of future research.

An appendix concerning quivers and Gabriel's theorem concludes the book.

The language of quivers is indeed quite powerful and the book has the great merit of presenting its application to persistence theory in a clear and manageable way. I am sure that it will be of use for people working at the boundary between homological algebra and topological data analysis. They will find in this book a useful summary of several new techniques that are presently available for research and applications. An addenda and errata webpage for the book is maintained at the link http://www.ams.org/bookpages/surv-209.

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Current version of review. Go to earlier version. *Patrizio Frosini*