For a data set S of n points the clique complex K_Σ is hard to compute when n is large. K_Σ has

- n vertices
- edges determined by the $\frac{1}{2}n(n-1)$ distances $d(x,y)$, $x,y \in S$
- 2-simplices determined by the $\frac{1}{6}n(n-1)(n-2)$ triples $x,y,z \in S$.

Typically $|S| > 1000$.

Mapper Clustering

Again based on the nerve $N\mathcal{U}$ of an open cover $\mathcal{U}$ of an unknown population space X.

To construct such a cover $\mathcal{U}$ we could choose:

1) a continuous function $f : X \to Z$ where Z is a <u>known</u> (parameter) space, and where we only know $f(x)$ for x in some <u>finite</u> sample $S \subset X$.

2) an open cover

$$\mathcal{V} = \{ V_\alpha \}_{\alpha \in A}$$

of Z.

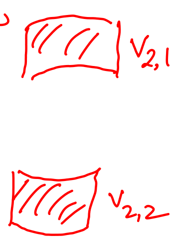
For $\alpha \in A$ the pre-image

$$U_\alpha = f^{-1}(V_\alpha)$$

is open since f is continuous. In general U_α may have many connected components.

In the example below

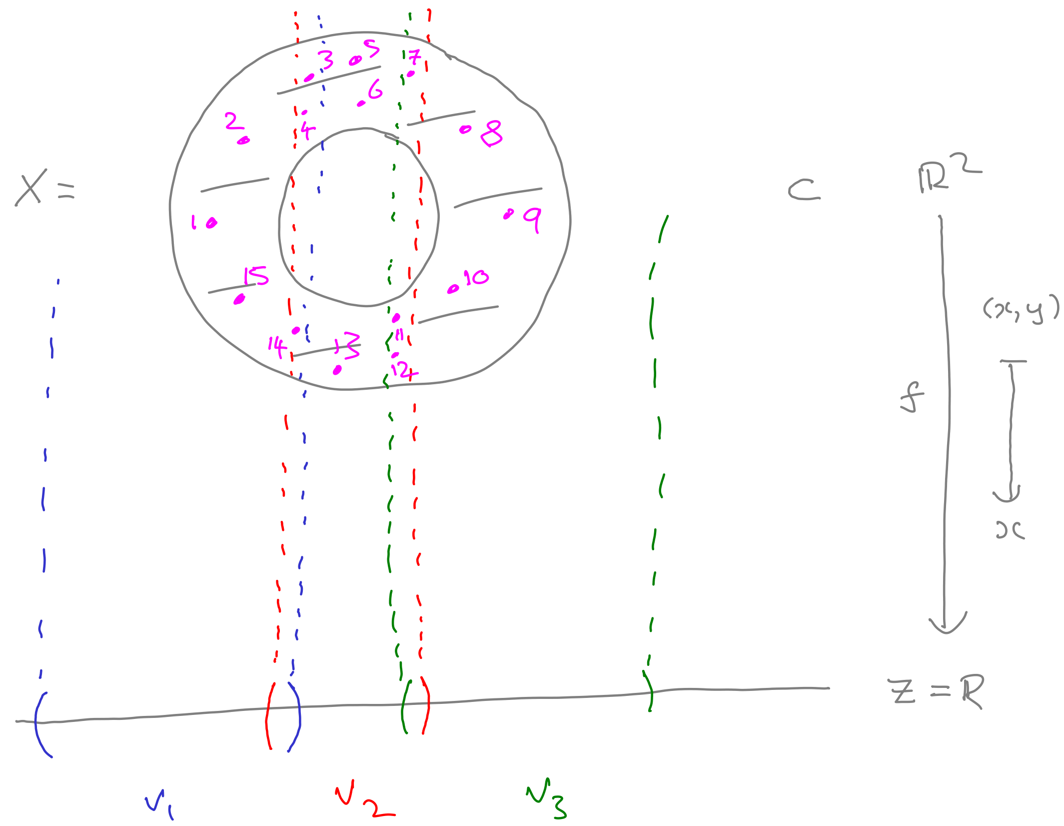
$$U_2 = f^{-1}(V_2)$$



In general

$$U_\alpha = U_{\alpha 1} \sqcup U_{\alpha 2} \sqcup \cdots \sqcup U_{\alpha n_\alpha}$$

with $U_{\alpha i}$ disjoint connected components.



$A = \{1, 2, 3\}$ $\qquad$ $\mathcal{V} = \{V_1, V_2, V_3\}$

$f^{-1}(V_1) \cap S = \{1, 2, 3, 4, 14, 15\} = S_{1,1}$

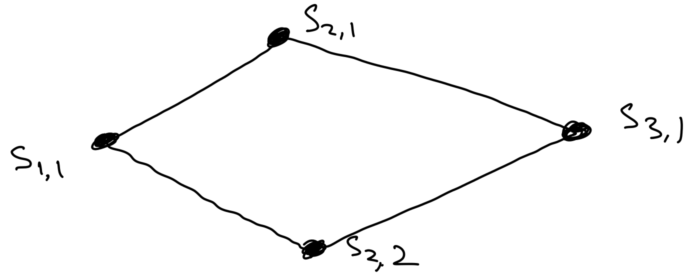
$f^{-1}(V_2) \cap S = \underbrace{\{3, 4, 5, 6, 7\}}_{S_{2,1}} \cup \underbrace{\{11, 12, 13, 14\}}_{S_{2,2}}$

$f^{-1}(V_3) \cap S = \underbrace{\{7, 8, 9, 10, 11, 12\}}_{S_{3,1}}$

So $S_{1,1} \cup S_{2,1} \cup S_{2,2} \cup S_{3,1} = S$,

or $\{S_{\alpha,i}\}$ is a cover of S.

The nerve $N\{S_{\alpha,i}\}$ is



Note: this nerve is homeomorphic to a circle S^1.

So we get an open cover

$$\mathcal{U} = \{ U_{\alpha,i} \}_{\substack{\alpha \in A \\ 1 \le i \le n_\alpha}}$$

of X.

The nerve $N\mathcal{U}$ is our "approximation" to X.

If f and $\mathcal{V}$ are well chosen we would hope to get $|N\mathcal{U}|$ homotopy equivalent to X.

<u>However</u> we can't construct the $U_{\alpha,i}$ since we don't know X. We approximate $U_{\alpha,i}$ by

$$f^{-1}(V_\alpha) \cap S = S_{\alpha,1} \sqcup S_{\alpha,2} \sqcup \cdots \sqcup S_{\alpha,n_\alpha}$$

$$\left(f^{-1}(V_\alpha) = U_{\alpha,1} \sqcup U_{\alpha,2} \sqcup \cdots \sqcup U_{\alpha,n_\alpha} \right)$$

Here the connected component $U_{\alpha,i}$ is "approximated" by a cluster $S_{\alpha,i}$ obtained by applying <u>any</u> clustering algorithm to $f^{-1}(V_\alpha) \cap S$.