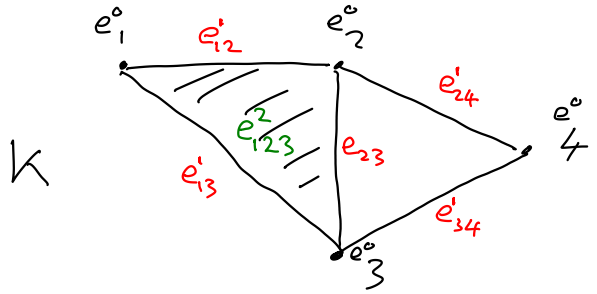

$$0 \longrightarrow C_2K \xrightarrow{d_2} C_1K \xrightarrow{d_1} C_0K \xrightarrow{d_0} 0$$

$$\mathbb{F} \qquad \mathbb{F}^5 \qquad \mathbb{F}^4$$

$$D_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$\mathbb{F} = \mathbb{Z}_2 = \{0, 1\}$

$d_1(e^1_{12}) = e^0_1 + e^0_2 + 0e^0_3 + 0e^0_4$

$d_1(e^1_{13}) = e^0_1 + e^0_3$

$d_1(e^1_{23}) = e^0_2 + e^0_3$

$d_1(e^1_{24}) = e^0_2 + e^0_4$

$d_1(e^1_{34}) = e^0_3 + e^0_4$

$d_2(e^2_{123}) = e^1_{12} + e^1_{13} + e^1_{23}$

$$H_1(C_*K) = \frac{\ker d_1}{\operatorname{im} d_2}$$

$$\dim(H_1(C_*K)) = \dim\left(\frac{\ker d_1}{\operatorname{im} d_2}\right) = \dim(\ker d_1) - \dim(\operatorname{im} d_2)$$

$$= \left(5 - \dim(\operatorname{im} d_1)\right) - \dim(\operatorname{im} d_2)$$

$$= (5 - \operatorname{rank} D_1) - \operatorname{rank} D_2$$

$$= (5 - 3) - 1$$

$$= 1$$

$H_1(C_*K) = \mathbb{F}$

$$H_0(C_*K) = \frac{\ker d_0}{\operatorname{im} d_1} = \frac{\mathbb{F}^4}{\operatorname{im} d_1}, \quad \dim(H_0(C_*K)) = 4 - 3 = 1$$

$H_0(C_*K) = \mathbb{F}$