$$\int_{\partial S} \omega = \int_S d\omega \qquad (*)$$

For $p=0$ and $n=1$, i.e, for a 0-form ω in 1 variable, we understand all terms in Stoke's formula $(*)$ except for $d\omega$

<u>Defn</u> For a differential 0-form $\omega = F(x)$ we define the 1-form

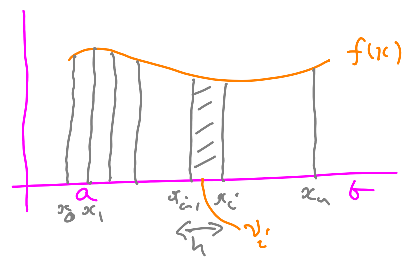
$$d\omega = F'(x)\,dx$$

we call $d\omega$ the total derivative of ω, or just derivative of ω.

For $p=0, n=1$ we see that $(*)$ is just the Fundamental Theorem of Calculus.

Let's recall from first year

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(\nu_i)\,(x_i - x_{i-1})$$



where

- $P = \{a = x_0, x_1, x_2, \ldots, x_n = b\}$, $a_i \in [a,b]$
- $\nu_i \in [x_{i-1}, x_i]$
- $\|P\| = \max_{1\le i\le n} |x_i - x_{i-1}|$

Proof of Fundamental Theorem of Calculus

Let's suppose that the Galway to Dublin train has a functioning speedometer, but a broken mileometer. The driver has a clock.

To estimate the distance travelled from time $t=a$ to time $t=b$ the driver could

$$\sum_{i=1}^{n} f(t_i)(t_i - t_{i-1})$$

where $f(t)$ is the speed of the train at time t, and $a = t_0 < t_1 < t_2 \cdots < t_n = b$

Let
$F(t) =$ be the total distance travelled at time t.

Now

$$f(t) = F'(t)$$

and roughly

$$F(b) - F(a) \simeq \sum_{i=1}^{n} f(v_i)(t_i - t_{i-1})$$

Taking limits as $\|P\| \to 0$

$$F(b) - F(a) = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(v_i)(t_i - t_{i-1})$$

Thus

$$\boxed{F(b) - F(a) = \int_a^b f(t)\, dt}$$

or

$$\boxed{\int_{\partial S} \omega = \int_S d\omega}$$

with $\omega = F(t)$, $S = [a, b]$

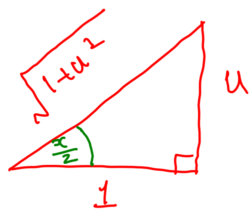
Example Find a differential 0-form ω whose total derivative is

$$d\omega = \frac{1}{5+3\cos(x)}\,dx .$$

Sol^n Using the language of 1st year maths, we want to find

$$\omega = \int \frac{1}{5+3\cos(x)}\,dx .$$

Let $u = \tan\left(\frac{x}{2}\right)$



$$\sin\left(\frac{x}{2}\right) = \frac{u}{\sqrt{1+u^2}}$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+u^2}}$$

$$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx \quad \left\{ \begin{aligned} dx &= 2\cos^2\left(\frac{x}{2}\right)du \\ &= \frac{2}{1+u^2}\,du \end{aligned} \right.$$

$$\begin{aligned} \cos(x) &= \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right) \\ &= \frac{1}{1+u^2} - \frac{u^2}{1+u^2} \\ &= \frac{1-u^2}{1+u^2} \end{aligned}$$

So

$$w = \int \frac{1}{5+3\cos(x)}\,dx$$

$$= \int \frac{1}{5+3\left(\frac{1-u^2}{1+u^2}\right)} \frac{2}{1+u^2}\,du$$

$$= \cdots$$

$$= \int \frac{1}{4+u^2}\,du$$

From Log book

$$w = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + C$$

$$\boxed{w = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\tan\left(\frac{x}{2}\right)\right) + C}$$