Example / Definition

Integrate the 0-form $\omega = 3x^2 + 4$ on ∂S where

$$S = [2,1] \cup [3,4]$$

Soln $\partial S = \{2, 3, 1, 4\}$

initial terminal

$$\int_{\partial S} \omega = \omega(1) + \omega(4) - \{\omega(2) + \omega(3)\}$$

↑ definition

$$= 7 + 52 - 16 - 31$$

$$= 12 \quad .$$

Background Reading:

"Advanced Calculus: a differential forms approach"
by Harold H. Edwards

Also: Spivak's book on
"Differential manifolds"

Stokes Formula

$$\int_{\partial S} \omega = \int_{S} d\omega$$

Case $n=1$ variables
$p=0$, ω is diff. 0-form

Differential 1-forms in 1 variable. ($p=1, n=1$)

A differential 1-form is a function of the type

$$\omega = f(x)\,h$$

which inputs two numbers $x, h \in \mathbb{R}$ and returns the number $f(x)\,h$, where $f(x)$ is some differentiable function.

Example Evaluate the 1-form

$$\omega = (x^2+6)\,h$$

at $x=2$, $h=0.5$

Soln $(2^2+6)(0.5) = 5$

Notation We usually denote the 1-form

$$f(x)\,h$$

by

$$f(x)\,dx$$

Example Evaluate the 1-form

$$\omega = \sin(x)\,dx$$

at $x = \frac{\pi}{2}$, $dx = 0.25$

Sol^n

$$\sin\left(\frac{\pi}{2}\right)(0.25) = 0.25$$

Defn Given a 1-form

$$\omega = f(x)\,dx$$

and an oriented interval

$$S = [a, b]$$

we define the integral as

$$\int_S \omega = \int_a^b f(x)\,dx$$

explained in 1st year

Informally: $\int_a^b f(x)\,dx$ is the area between the curve $y = f(x)$ and the x-axis from a to b where, if $a < b$, areas above the x-axis are positive and areas below the x-axis are considered negative.

Problem: A fundraising project has daily expenditure of $\$10\,000$. The rate of contributions at time t is modelled by

$$C(t) = -100t^2 + 20\,000.$$

What net proceeds can be expected from the project?

Soln Project runs until

$$C(t) \leq 10\,000$$

$$-100t^2 + 20000 = 10000$$
$$100t^2 = 10000$$
$$t = 10$$

The project will run over the oriented interval

$$S = [0, 10].$$

Contributions are modelled by the 1-form

$$u = (-100t^2 + 20000)\,dt$$

Expenditure is modelled by the 1-form

$$v = -10\,000\,dt$$

The net rate of income is modelled by the 1-form

$$w = u + v = (-100t^2 + 10\,000)\,dt$$

The project can be expected to make

$$\int_S w$$

where $S = [0, 10]$.

$$\int_S w \overset{\text{defn}}{=} \int_0^{10} (-100t^2 + 10000)\, dt$$

$$= -\frac{100t^3}{3} + 10000t \Big|_0^{10}$$

$$= \$\ 66\,666.67$$