Problem: you intercept the ciphertext

OH7F86BB46R36270268B9 ciphertext

and you know: $\phi\phi 7$ plaintext

1) A 37-letter alphabet is used

$\phi, 1, 2, 3, 4, 5, 6, 7, 8, 9, A=10, B=11, \cdots, Z=35, \_=36$

2) An affine cryptosystem

$$X \longmapsto \alpha X + \beta \mod 37$$

is used on single letter message units with enciphering key $E = (\alpha, \beta)$

3) plaintext ends $\phi\phi 7$

Enciphering function

$X \longmapsto \alpha X + \beta \mod 37$

$\phi \longmapsto \alpha \cdot \phi + \beta = \beta$

$\phi \longmapsto \boxed{\beta = 11}$

$7 \longmapsto 7\alpha + \beta = 9$

$7\alpha + \beta = 9 \mod 37$

$\beta = 11$

$7\alpha = -2$

$\alpha = 7^{-1}(-2) \mod 37$

To find $7^{-1} \mod 37$ we use the Euclidean algorithm.

$37 = 5 \cdot 7 + 2$

$7 = 3 \cdot 2 + 1 \longleftarrow \gcd(7, 37)$

$1 = 7 - 3 \cdot 2$

$= 7 - 3(37 - 5 \cdot 7)$

$= 16 \cdot 7 - 3 \cdot 37$

$\equiv 16 \cdot 7 \mod 37$

So $\boxed{7^{-1} \equiv 16 \mod 37}$

$\alpha = 7^{-1}(-2) \equiv 16(-2) \equiv -32 \equiv 5$

$\boxed{\alpha = 5, \ \beta = 11}$

Deciphering function

$Y \longmapsto 5^{-1}(Y-11) \mod 37$

what is $5^{-1} \mod 37$

$37 = 7.5 + 2$

$5 = 2.2 + 1$

$1 = 5 - 2.2$

$= 5 - 2(37 - 7.5)$

$= 15.5 \mod 37$

$\boxed{5^{-1} \equiv 15 \mod 37}$

Deciphering function:

$Y \longmapsto 5^{-1}(Y-11) \mod 37$

$\equiv 15(Y-11) \mod 37$

$\equiv 15Y - 165 \mod 37$

$\equiv 15Y - 17$

$\equiv 15Y + 20$

$\boxed{-17 \equiv +20 \mod 37}$

Deciphering key $D = (15, 20)$.

Now we need to decipher the ciphertext, one letter at a time.

$O \longmapsto 15.O + 20 \mod 37$

$O = 24$

$24 \longmapsto 15.24 + 20 \mod 37$

$= 360 + 20 \mod 37$

$= 380 \mod 37$

$\equiv 10 \mod 37$

$= A$

240
120
360