Matrix Multiplication

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = B$$

- who cares ?
- How did h get B from A ?

Who cares ?

$$\left.\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 5y + 5z &= 2 \\ 3x + 8y + 6z &= 3 \end{aligned}\right\} \quad (*)$$

The system of equations $(*)$ can be re-written as follows :

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad (**)$$

To find x, y, z we could multiply both sides of $(**)$ by A^{-1} to get:

$$A^{-1} A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad (***)$$

From above we get

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Hence $x = 1$, $y = 0$, $z = 0$.

Where did B come from ?

Gauss - Jordan method for finding the inverse of an $n \times n$ matrix A

The idea is

$$\underset{n \times 2n}{\left(A \mid I \right)} \xrightarrow[\text{row operations}]{\text{elementary}} \left(I \mid B \right)$$

where $B = A^{-1}$.

Elementary row operations :

(I) $R_i \leftarrow R_i + \lambda R_j$ $\qquad j \neq i, \ \lambda \in \mathbb{R}$

(II) $R_i \leftarrow \lambda R_i$ $\qquad 0 \neq \lambda \in \mathbb{R}$

(III) $R_i \leftrightarrow R_j$ $\qquad i \neq j$

Let's find A^{-1} where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}.$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -3 & 0 & 1 \end{array}\right) \qquad \begin{array}{l} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 3R_1 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right) \qquad R_3 \leftarrow R_3 - 2R_2$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right) \qquad R_3 \leftarrow -1 . R_3$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & -6 & 3 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right) \qquad \begin{array}{l} R_1 \leftarrow R_1 - 3R_3 \\ R_2 \leftarrow R_2 + R_3 \end{array}$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{matrix}}_{B} \\ 0 & 0 & 1 & \end{array}\right) \qquad R_1 \leftarrow R_1 - 2R_2$$

So

$$A^{-1} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}.$$

Tomorrow: we'll see why this method works.