MA180 / MA185 / MA190 Mathematics (Algebra & Calculus)

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Algebra

Topics	Context
• Elementary number theory	Internet communication
• Matrix theory	Geometry & internet communication
• Eigenvalues & Eigenvectors	Breeding Rabbits & Google search engine

Elementary Number Theory

$5 + 15 = 20$ in school

$9 + 18 = 3$ on a 12-hour clock

$9 + 18 \equiv 3 \mod 12$

Today is Monday. In 73 days time it will be Thursday.

$1 + 73 \equiv 4 \mod 7$

More examples

$10 \times 5 \equiv 2 \mod 12$

$7 + 5 \equiv 3 \mod 9$

$7 \times 8 \equiv 2 \mod 9$

$2 - 5 \equiv 5 \mod 8$

what is $\frac{1}{3}$?

Daniel says:

$\frac{1}{3}$ is that number with the property that

$$\left(\frac{1}{3}\right) \times 3 = 1$$

Alternative notation

$3^{-1} = \frac{1}{3}$ We call 3^{-1} the multiplicative inverse of 3.

Back to clocks

What is $7^{-1} \mod 10$?

$$7^{-1} \equiv 3 \mod 10$$

because

$$7 \times 3 \equiv 1 \mod 10$$

Applications

Any book is identified by its ISBN. On older books this is a string of 10 digits.

0 3 4 0 6 8 1 0 6 3

is the number for fire on a treasure island.

The final digit is a safety check digit. Because

$(1\times 0)+(2\times 3)+(3\times 4)+(4\times 0)+(5\times 6)+(6\times 8)+(7\times 1)+(8\times 0)+(9\times 6)+(10\times 3)$

$= 0+6+1+0+8+4+7+0+10+8$

$\equiv 0 \mod 11$