Calculus

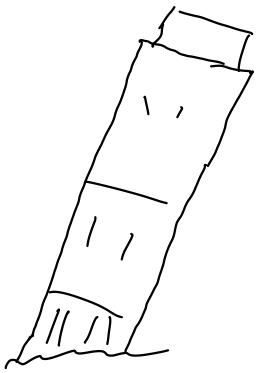
Topics

1) Functions & limits

2) Rates of change

3) Differential Equations

Quick introduction to (1) & (2)



Pisa

Stone falls y metres in t seconds

Experiment suggests

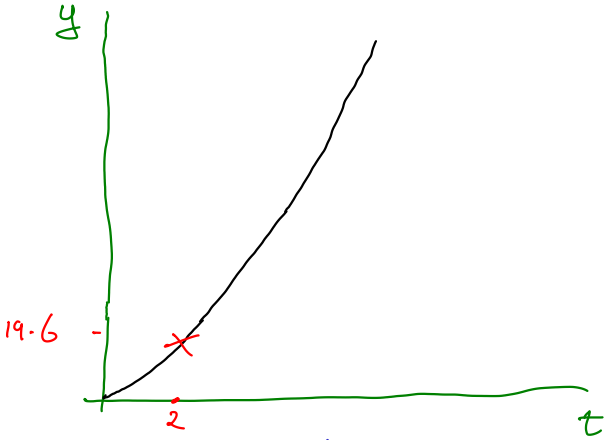
$$y = 4.9\,t^2$$

m (=metres)

We say **y is a function of t**

This means that for each of t there is one corresponding value of y.

Functions are represented by their graph.



Question What is the average speed of the stone between $t=2$ and $t=3$ second?

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time}}$$

$$= \frac{y(3) - y(2)}{1}$$

$$= \frac{4.9(3^2 - 2^2)}{1} = 4.9 \times 5 = 24.5 \text{ m/s}$$

Question what is the speed of the stone at time $t = 2$ second ?

The speed at $t=2$ is

$$v(2) = \lim_{h \to 0} \frac{y(2+h) - y(2)}{h}$$

$$= \lim_{h \to 0} \frac{4.9(2+h)^2 - 4.9(2^2)}{h}$$

$$= \lim_{h \to 0} \frac{4.9(4 + 4h + h^2 - 4)}{h}$$

$$= \lim_{h \to 0} \frac{4.9(4h + h^2)}{h} = \lim_{h \to 0} \frac{4.9 \cancel{h}(4+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 4.9(4+h)$$

$$= 19.6 \quad m/sec$$

Consider

$$f(x) = \frac{\cos(3x\pi)}{2} + \frac{\cos(3^2 x\pi)}{2^2} + \frac{\cos(3^3 x\pi)}{2^3} + \cdots$$

$$f(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{\cos(3^k x\pi)}{2^k}$$

Karl Weierstrass (1872) explained why $f(x)$ is everywhere continuous but differentiable nowhere.

Émil Picard: Newton would not have developed the calculus if he had known about Weierstrass's function.

The function $f(x)$ contradicted previous "results" of Ampère who used vague definitions.

Our aim: understand calculus rigorously.