3b) i) $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(3-\lambda) - 1.2 = 0$$

$$\lambda^2 - 7\lambda + 10 = 0$$

$$(\lambda - 5)(\lambda - 2) = 0$$

Eigenvalues are $\lambda_1 = 5$, $\lambda_2 = 2$.

ii) $D = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$

$E = \begin{pmatrix} | & | \\ v_1 & v_2 \\ | & | \end{pmatrix}$

v_1, v_2 eigenvectors for λ_1, λ_2.

<u>Case $\lambda_1 = 5$</u>

$$\begin{pmatrix} 4-5 & 2 \\ 1 & 3-5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Take $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

<u>Case $\lambda_2 = 2$</u>

$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Take $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

$E = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$.

For part (a):
Standard basis vectors for $\mathbb{R}^2$ are $e_1 = (1,0)$, $e_2 = (0,1)$.

$\frac{d}{dt} \cos(t^2) = -\sin(t^2) \cdot 2t$

$\frac{d}{dt}\left(-\frac{1}{2}\cos(t^2)\right) = t\sin(t^2)$

$\frac{d}{dt}(\ln(x)) = \frac{1}{x}$

$\frac{d}{dx} \ln(\sin(t^2)) = \frac{\cos(t^2) \cdot 2t}{\sin(t^2)}$

6a) Anti-derivative of $f(t) = t\sin(t^2)$ is

$-\frac{1}{2}\cos(t^2)$

Anti-derivative of $g(t) = \dfrac{t\cos(t^2)}{\sin(t^2)}$ is

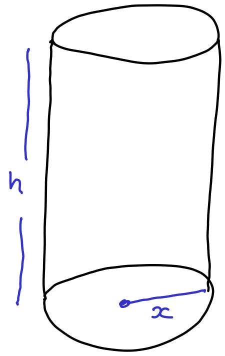
$\frac{1}{2}\ln(\sin(t^2))$

Anti-derivative of $(t+3)^{\frac{1}{2}}$ is

$\frac{2}{3}(t+3)^{\frac{3}{2}}$.

6b) Done in Lecture 43.

5a)



Surface area of container is

$A = 2\pi x h + \pi x^2 = 6\pi$

$2\pi x h = 6\pi - \pi x^2$

$h = \dfrac{6\pi - \pi x^2}{2\pi x}$

$h = \dfrac{6 - x^2}{2x}$

$h = \dfrac{3}{x} - \dfrac{x}{2}$

Volume is $V = \pi x^2 h$

$V = \pi x^2 \left(\dfrac{3}{x} - \dfrac{x}{2}\right) = 3\pi x - \dfrac{\pi x^3}{2}$

$\dfrac{dV}{dx} = 3\pi - \dfrac{3\pi x^2}{2} = 3\pi\left(1 - \dfrac{x^2}{2}\right) = 0$

So $\dfrac{x^2}{2} = 1$ or $x^2 = 2$ or $\boxed{x = \sqrt{2}}$

Answer

5b) i) $f'(x) = 3x^2 - 6x - 24$ $f''(x) = 6x - 6 = 6(x-1)$

$f'(x) = 3(x^2 - 2x - 8)$

$f'(x) = 3(x-4)(x+2)$

Critical points occur when $f'(x) = 0$,

that is when $x = 4$ and $x = -2$.

i.e. $(4, -74)$, $(-2, 34)$

x		-2				4	
f'	$+$	0	$-$		$-$	0	$+$

So $(-2, 34)$ is a local max.

So $(4, -74)$ is a local min.

iii) A point of inflection is a point where the concavity changes

iv) Point of inflection occurs at $x = 1$.