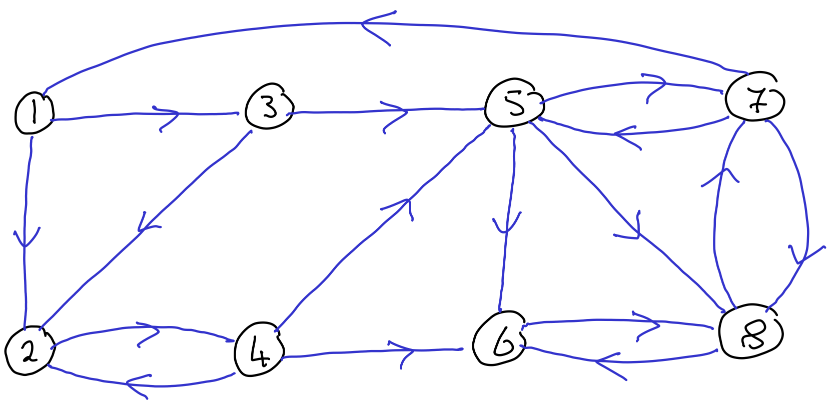
Google

A list of words
Matrices, eigenvalues, eigenvectors
results in a few pages being listed
as most likely of interest.
The web pages containing the words can
be represented as a diagram of nodes
(one node for each WWW page) and
arrows (corresponding to a link from one
page to another).



When listing pages Google first assigns a
number I_n to each page P_n. This I_n
is the "importance" of page P_n. Google lists
the most important page first.

$$
\left.
\begin{aligned}
I_1 &= \frac{I_7}{3} \\
I_2 &= \frac{I_1}{2} + \frac{I_3}{2} + \frac{I_4}{3} \\
I_3 &= \frac{I_1}{2} \\
I_4 &= I_2 \\
I_5 &= \frac{I_3}{2} + \frac{I_4}{3} + \frac{I_7}{3} \\
I_6 &= \frac{I_4}{3} + \frac{I_5}{3} + \frac{I_8}{2} \\
I_7 &= \frac{I_5}{3} + \frac{I_8}{2} \\
I_8 &= \frac{I_5}{3} + I_6 + \frac{I_7}{3}
\end{aligned}
\right\}
\text{System of linear equations}
$$

How do we determine the I_n?

Let's express this system using matrices.

$$\underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 1 & \frac{1}{3} & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{pmatrix}}_{v} = \underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{pmatrix}}_{v}$$

Note: v is an eigenvector of A with corresponding eigenvalue $\lambda = 1$.

using GAP and vector for A with $\lambda = 1$ is

Google lists pages in the following order:

$$v = \begin{pmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{pmatrix} \begin{matrix} \\ \times \\ \\ \times \\ \\ \\ \\ \\ \end{matrix}$$

P_8
P_6
P_7
P_5
P_2
P_4
P_1
P_3

But: how do we calculate the eigenvectors of a square matrix A.