First quiz: 10 am Thu 3 December

Those with a lab at that time: Friday 2 pm instead.

Quiz: 50 minutes

Okusou: open at 10.00 am, close at 10.50 am

First quiz covers:

Q1, Q2, Q4

algebra (Q1, Q2)

Calculus (Q4)

2019-20

Q 2a)

$$\begin{pmatrix} N \\ O \end{pmatrix} \sim \begin{pmatrix} 13 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} O \\ N \end{pmatrix} \sim \begin{pmatrix} 14 \\ 13 \end{pmatrix}$$

$$f\begin{pmatrix} 13 \\ 14 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 13 \\ 14 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 26 + 14 + 4.14 \\ 13 + 3.14 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} \qquad \text{mod } 29$$

$$= \begin{pmatrix} 9 \\ -3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ 28 \end{pmatrix} = \begin{pmatrix} O \\ \_ \end{pmatrix}$$

$$f\begin{pmatrix} 14 \\ 13 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 14 \\ 13 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 - 6 + 13 \\ 14 + 13 - 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 11 \\ 26 \end{pmatrix} = \begin{pmatrix} L \\ : \end{pmatrix}$$

Enciphered message is

O _ L :

Q3 a)

$$f\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix} \qquad g\begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$f\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} \qquad g\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\underset{F}{\begin{pmatrix}2 & 1\\4 & 3\end{pmatrix}}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$$

$$\underset{F}{\begin{pmatrix}2 & 1\\4 & 3\end{pmatrix}}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$$

$F = \begin{pmatrix}2 & 1\\4 & 3\end{pmatrix}$ is the matrix of f.

The matrix of g is $F^{-1} = \begin{pmatrix}2 & 1\\4 & 3\end{pmatrix}^{-1} = \dfrac{1}{2.3-1.4}\begin{pmatrix}3 & -1\\-4 & 2\end{pmatrix}$

Matrix of g is $\dfrac{1}{2}\begin{pmatrix}3 & -1\\-4 & 2\end{pmatrix} = \begin{pmatrix}\frac{3}{2} & -\frac{1}{2}\\-2 & 1\end{pmatrix}$.

$$\mathbb{R}^2 \xrightarrow{f} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}^2$$

The matrix of $f \circ f$ is

$$FF = \begin{pmatrix}2 & 1\\4 & 3\end{pmatrix}\begin{pmatrix}2 & 1\\4 & 3\end{pmatrix} = \begin{pmatrix}8 & 7\\20 & 13\end{pmatrix}$$

Q4 a)(i) $\lim_{x \to 1} \frac{x^5 - 1}{x - 1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^4 + x^3 + x^2 + x + 1)}{\cancel{x-1}}$

$= \lim_{x \to 1} x^4 + x^3 + x^2 + x + 1 = 5$

Q4 a) (ii) $\lim_{\theta \to \frac{\pi}{2}} \frac{\cos^2 \theta}{(1 - \sin \theta)} = \lim_{\theta \to \frac{\pi}{2}} \frac{\cos^2 \theta (1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$

$= \lim_{\theta \to \frac{\pi}{2}} \frac{\cancel{\cos^2 \theta} (1 + \sin \theta)}{\cancel{(1 - \sin^2 \theta)}}$

$= \lim_{\theta \to \frac{\pi}{2}} 1 + \sin \theta = \underline{\underline{2}}$

Q4 h) $x^3 + x = -1$.

$x^3 + x + 1 = 0$. $(*)$

Let $f(x) = x^3 + x + 1$.

Note that $f(x)$ is continuous on $\mathbb{R}$.

$f(0) > 0$

$f(-1) < 0$.

The IVT says that there is <u>at</u> least one x satisfying equation $(*)$.

$f'(x) = 3x^2 + 1$.

So $f'(x) > 0$ for all x.

If $f(x) = 0$ for two distinct values of x then Rolle's Theorem says $f'(c) = 0$ for some c. Hence $f(x)$ has at most one root.