10 am Wed 16 December - algebra quiz - 3 questions

10 am Thus 17 December - Calculus quiz - 3 questions

No surprises! Each quiz: 1 hour + 15 mins for scanning & uploading (upload a single pdf file: Office Lens)

Sem I

40% quizzes

40% CA

20% projects

2019-20 Q1

a(i)

$29 = 16 + 13$

$16 = 13 + 3$

$13 = 4.3 + 1$

$1 = 13 - 4.3$

$= 13 - 4(16-13) = 5.13 - 4.16$

$= 5(29-16) - 4.16 = 5.29 - 9.16$

$\equiv -9.16 \mod 29$

$\equiv 20.16$

Hence $16^{-1} \equiv 20 \mod 29$

a(ii) $f_E : \mathbb{Z}_{29} \to \mathbb{Z}_{29}, \quad x \mapsto 16x + 4$

$f_D : \mathbb{Z}_{29} \to \mathbb{Z}_{29}, \quad x \mapsto 16^{-1}(x-4) \mod 29$

$x \mapsto 20(x-4) \mod 29$

$x \mapsto 20x - 22 \mod 29$

$x \mapsto 20x + 7 \mod 29$

$x \mapsto -9x + 7$

DO NOT USE A RED PEN IN YOUR ANSWERS

a(iii)

N Z Z X

13, 25, 25, 23

$13 \mapsto -9.13 + 7 = 6 \sim G \mod 29$

$25 \mapsto -9.25 + 7 = 9.4 + 7 = 14 \sim O$

$-9.13 + 7 \quad -117 + 7 = -110 = 6$

$116 = 0 \quad 110 = -6$

Answer is GOOD

b(i)

$4 \overline{)2520}$ 630

24

120

$2520 = 4.630$

$= 8.315$

$= 8.3.105$

$= 8.9.35$

$= 8.9.5.7$

$= 2^3.3^2.5.7$

b(ii)

$\phi(2520) = \phi(2^3.3^2.5.7)$

$= \phi(2^3)\,\phi(3^2)\,\phi(5)\,\phi(7)$

$= (2^3-2^2)(3^2-3)(5-1)(7-1)$

$= 4.6.4.6$

$= 576$

b (iii)

$11^{578} = 11^{576} \cdot 11^2 \equiv 11^2 \equiv 121 \mod 2520$

We know $11^{\phi(2520)} \equiv 1 \mod 2520$

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2 (b)

$$\left(\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right) \quad R_3 \to R_3 - 2R_1$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -4 & -1 & -2 & 0 & 1 \end{array}\right) \quad R_3 \to R_3 + 4R_2$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 4 & 1 \end{array}\right) \quad R_3 \to -R_3$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -1 \end{array}\right) \quad R_1 \to R_1 - 2R_3$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 0 & -3 & 8 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -1 \end{array}\right) \quad R_1 \to R_1 - 4R_2$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 4 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -3 & 4 & 2 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{pmatrix}$$