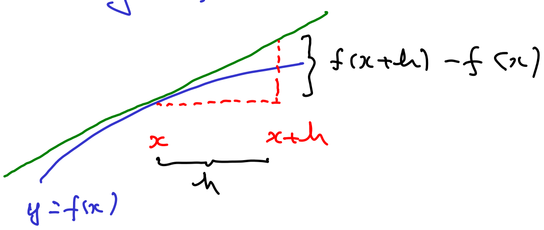
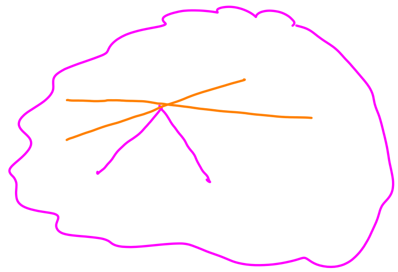
# Applications II

Applications where the derivative is used to measure the slope of a tangent.

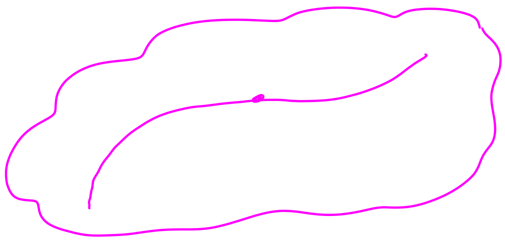



$$\text{Slope of tangent} \approx \frac{f(x+h)-f(x)}{h}$$

At points where a continuous function f(x) is a local maximum or local minimum

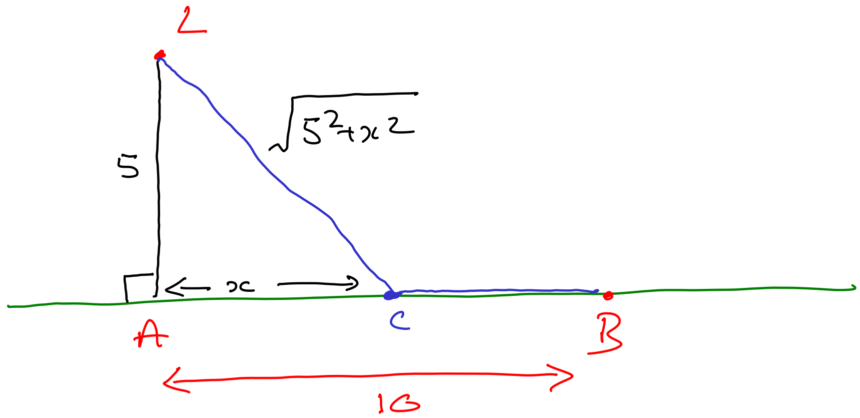



we have that the derivative $f'(x) = 0$ or else $f'(x)$ does not exist.



**Problem** A light house $L$ is located on a small island 5 km North of a point $A$ on a straight east-west coastline. A cable is to be laid from $L$ to a point $B$ on the coastline 10 km east of $A$. Laying the cable under water costs € 5000 per kilometer. Laying it on land costs € 3000 per kilometer.

**Question** What is the cheapest cost of laying the cable?




$x$ = distance from $A$ to $C$.

Let $f(x)$ = cost of laying the (blue) cable.

$$f(x) = 5000\sqrt{5^2+x^2} + 3000(10-x)$$

$$f(x) = 5000(5^2+x^2)^{\frac{1}{2}} + 3000(10-x)$$

$$f'(x) = \frac{5000}{\cancel{2}}(5^2+x^2)^{-\frac{1}{2}}\cancel{2}x - 3000$$

$$f'(x) = \frac{5000x}{\sqrt{5^2+x^2}} - 3000$$

$f'(x)$ exists for all $x \in \mathbb{R}$.

Now let's find when $f'(x) = 0$.

$$0 = \frac{5000x}{\sqrt{5^2+x^2}} - 3000$$

$$3000 = \frac{5000x}{\sqrt{5^2+x^2}}$$

$$3 = \frac{5x}{\sqrt{5^2+x^2}}$$

$$3\sqrt{5^2+x^2} = 5x$$

$$9(5^2+x^2) = 25x^2$$

$$9.5^2 = 16x^2$$

$$x^2 = \frac{3^2 . 5^2}{4^2}$$

$$x = \pm \frac{15}{4}.$$

So $f'(x) = 0$ when $x = \frac{15}{4}$.

Common sense tells us that the minimum cost occurs when $x = \frac{15}{4}$.

The minimum cost of laying the cable is

$$\boxed{f\left(\frac{15}{4}\right) = 5000\sqrt{5^2 + \frac{15^2}{4^2}} + 3000\left(10 - \frac{15}{4}\right) \text{ euro.}}$$

Min cost is €50000.