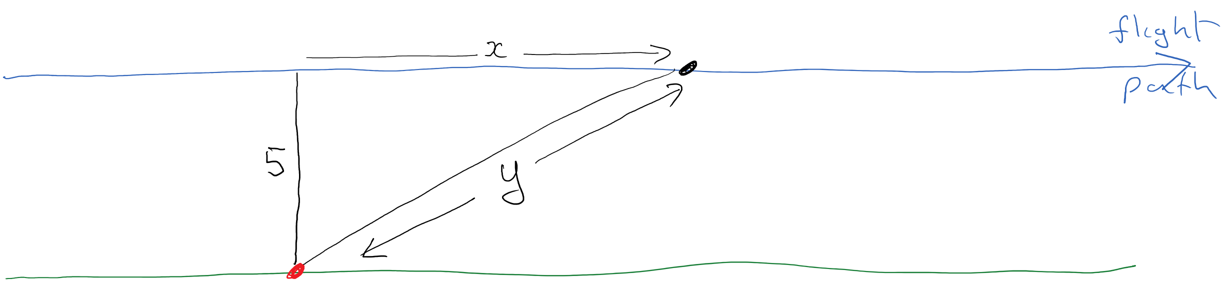
The real stuff : Applications

The derivative of a function can be thought of as a rate of change.

Problem

An aircraft is flying horizontally at a speed of 600 km/h. How fast is the distance between the aircraft and a radio beacon increasing 1 minute after the plane passes 5 km above the beacon?



Want to find $\boxed{\dfrac{dy}{dt} \text{ when } t=1}$

we know $\boxed{\dfrac{dx}{dt} = 600 \text{ km/h}}$

$= 10 \text{ km/m}$

When $t=1$ we have $x = 10$.

$$\boxed{x^2 + 5^2 = y^2 \qquad (*)}$$ for all t.

$$\boxed{\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \quad \text{Chain Rule}}$$

Differentiate both sides of $(*)$ with respect to t.

$$\boxed{2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt}} \qquad (**)$$

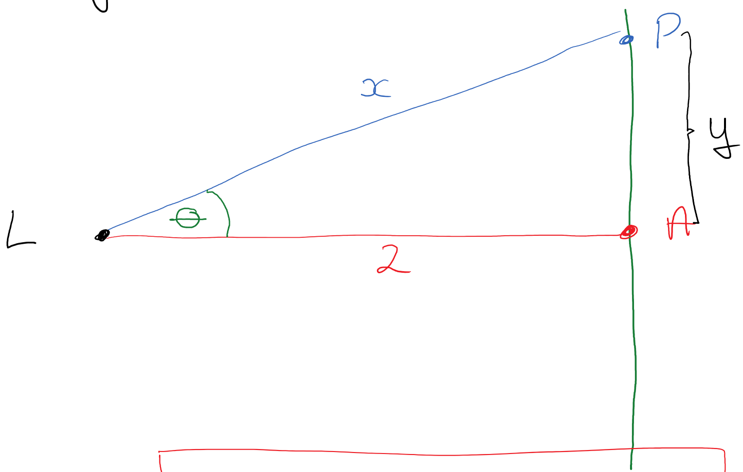
When $t=1$ equation $(**)$ becomes

$$2 . 10 . 10 = 2\sqrt{10^2 + 5^2} \; \frac{dy}{dt}$$

answer

$$\frac{dy}{dt} = \frac{200}{2\sqrt{125}} = \frac{100}{\sqrt{5.25}} = \frac{100}{\sqrt{5}\sqrt{25}} = \frac{20}{\sqrt{5}} \text{ km/min}$$

<u>Problem</u> A lighthouse L is located on a small island 2 km from the nearest point A on a long straight shore line. The lighthouse light rotates at 3 revs per minute. How fast is the illuminated spot P on the shoreline moving when it is 4 km from A?



want to find $\boxed{\dfrac{dy}{dt} \text{ when } y = 4.}$

$$\boxed{\frac{d\theta}{dt} = 6\pi}$$

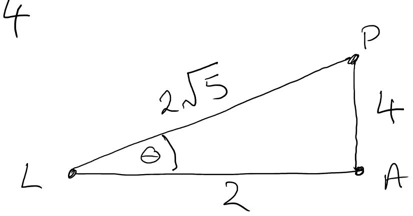
Both θ and y are functions of t.

$$\boxed{\tan\theta = \frac{y}{2}} \qquad (*)$$

Differentiate both sides of $(*)$ with respect to t.

$$\boxed{\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt}} \qquad (**)$$

when $y=4$



$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{2\sqrt{5}}{2} \quad \text{when } y=4.$$

$$= \sqrt{5}$$

from $(**)$, when $y = 4$,

$$\frac{dy}{dt} = 2 \cdot 5 \cdot 6\pi = \boxed{60\pi \text{ km/min}}$$

answer