$$\begin{aligned} \boxed{2}x + 3y + 2z &= 100 \\ x + y + 4z &= 70 \\ 20x + 10y + 10z &= 500 \end{aligned}$$

System of linear equations

This system is equivalent to the following system.

$$\left[\begin{array}{l} R_2 \leftarrow R_2 - \tfrac{1}{2} R_1 \\ R_3 \leftarrow R_3 - 10 R_1 \end{array}\right]$$

$$\begin{aligned} 2x + 3y + 2z &= 100 \\ \boxed{-\tfrac{1}{2}}y + 3z &= 20 \\ -20y - 10z &= -500 \end{aligned}$$

Second system of equations

This second system of equations is equivalent to the following system.

$$[\, R_3 \leftarrow R_3 - 40 R_2 \,]$$

$$\begin{aligned} 2x + 3y + 2z &= 100 \\ -\tfrac{1}{2}y + 3z &= 20 \\ -130z &= -1300 \end{aligned}$$

$$\begin{aligned} 0x + y + z &= 10 \\ 2x + y + 3z &= 4 \\ 2x + 2y + z &= 12 \end{aligned}$$

Back substitution:

$z = 10$

$y = 20$

$x = 10$

$$\begin{aligned} x + 2y + z &= 1 \\ x + 2y + z &= 2 \\ x + 2y + 3z &= 25 \end{aligned}$$

Terminology

2 is the pivot in the first stage.

$-\frac{1}{2}$ is the pivot in the second stage.

The above procedure for solving a system of linear equations is called gaussian elimination.

Q) Does this procedure always work?

A) No. It doesn't work if a pivot at some stage is zero. Some systems have no solution and so the procedure clearly can't work on these.

Back to Calculus

Rates of Change and Differentiation

Given a function $f(x)$ we define its <u>derivative</u> to be a function $f'(x)$ defined as

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} .$$

Example Find the derivative $f'(x)$ of the function $f(x) = x^2$.

Sol^n

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{(2x+h)\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x+h)$$

$$= 2x .$$

So for $f(x) = x^2$ we find

$f'(x) = 2x$.

Notation For $y = f(x)$ we'll often write

$$\frac{dy}{dx}$$

instead of

$$f'(x) .$$

- $\frac{d}{dx} x^n = n x^{n-1}$ for any $n \neq 0$.
- $\frac{d}{dx} \sin x = \cos x$
- $\frac{d}{dx} \cos x = -\sin x$
- $e = 2.71 \cdot \text{-----}$

  $\frac{d}{dx} e^x = e^x$
- etc.