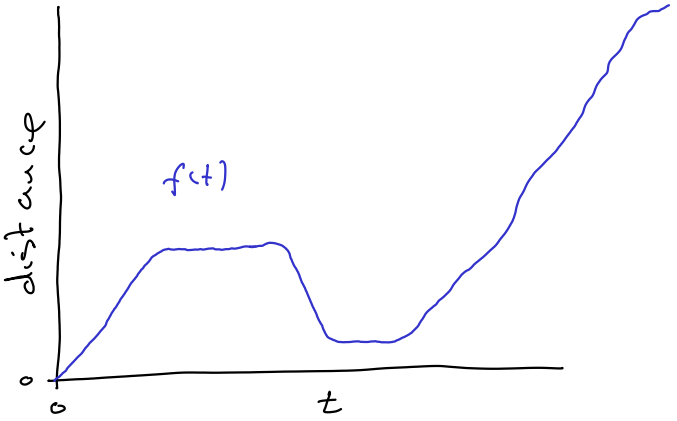
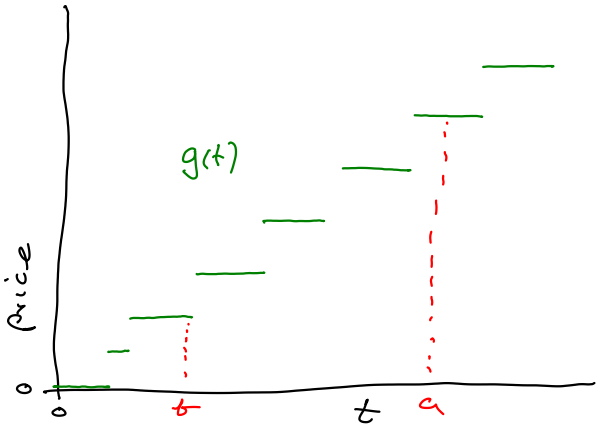
Continuity

h travel to Dublin airport.

$f(t)$ = distance from Galway t minutes after leaving Galway.

$g(t)$ = price of parking my car t minutes after entering Dublin Airport carpark.





Intuitively: Continuous means there are no breaks in the graph.

$f(t)$ is continuous

$g(t)$ is not continuous

Better, more general definition:

> A function $f(t)$ is continuous if a "small" change in the input t yields only a "small" change in the output $f(t)$.

In the context of functions

$f: \mathbb{R} \longrightarrow \mathbb{R}$

the following is the best definition.

We say that $f(t)$ is <u>continuous</u> at a point $t = a$ if:

i) $f(a)$ is defined (i.e. a must lie in the domain of f)

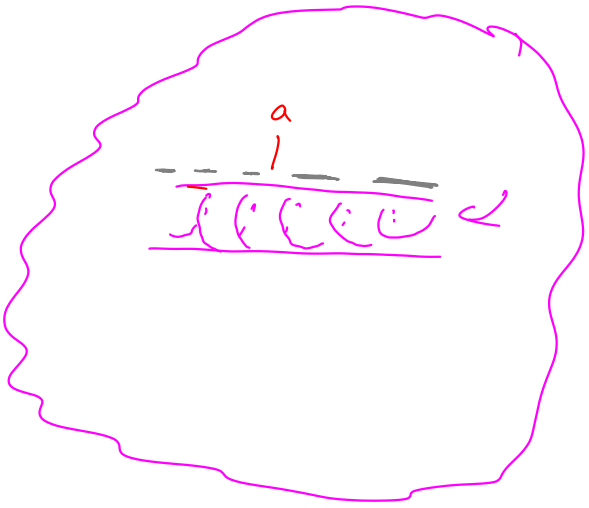
ii) $\lim_{t \to a} f(t)$ exists, and

iii) $\lim_{t \to a} f(t) = f(a)$.

Example Determine the constant k such that

$$f(x) = \begin{cases} x^3, & \text{for } x \geq 2 \\ kx, & \text{for } x < 2 \end{cases}$$

is continuous at all point $x \in \mathbb{R}$.



Sol^{n} The only problem is at $x = 2$.

We need

$$\lim_{x \to 2} f(x) = f(2) = 8$$

i.e. we need

$$\lim_{x \to 2-} f(x) = 8 = \lim_{x \to 2+} f(x)$$

i.e. need

$$\lim_{x \to 2-} kx = 8 = \lim_{x \to 2+} x^3$$

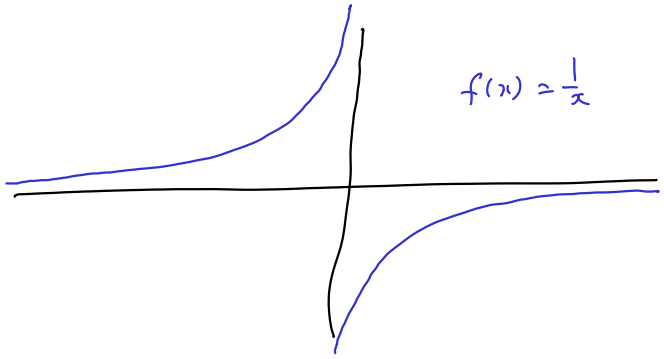
So we need

$$k \cdot 2 = 8$$

So we need $k = 4$.

Defn A function f is **continuous** if it is continuous at all points x in its domain.

Example Is the function $f(x) = \frac{1}{x}$ continuous?



Domain$(f) = \mathbb{R} \setminus \{0\}$.

Yes, f is continuous.