

Consider  $g(x) = \frac{x^6 - 1}{x - 1}$

Codomain  $(g) = \mathbb{R}$

Domain  $(g) = \mathbb{R} \setminus \{1\}$

$$g(0.9) = \frac{(0.9)^6 - 1}{0.9 - 1} = 4.68559$$

$$g(1.1) = \frac{(1.1)^6 - 1}{1.1 - 1} = 7.71561$$

$$g(0.99) = \frac{0.99^6 - 1}{0.99 - 1} = 5.85198506$$

$$g(0.999) = \frac{0.999^6 - 1}{0.999 - 1} = 5.985019985$$

We write

$$\lim_{x \rightarrow 1} g(x) = 6$$

to mean that for all real numbers  $x$  "close to 1", BUT DISTINCT FROM 1, the value of  $g(x)$  gets "close to 6"

## Example Evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x}$$

Sol<sup>n</sup> for  $x \neq 0$  and close to 0

$$\frac{\sqrt{4+x} - 2}{x} \cdot \frac{(\sqrt{4+x} + 2)}{(\sqrt{4+x} + 2)}$$

$$= \frac{4+x - 4}{x(\sqrt{4+x} + 2)}$$

$$= \frac{\cancel{x}}{\cancel{x}(\sqrt{4+x} + 2)} = \frac{1}{\sqrt{4+x} + 2}$$

So

$$\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

Example Evaluate

$$\lim_{x \rightarrow 2} \left( \frac{1}{x-2} - \frac{4}{x^2-4} \right)$$

Sol<sup>n</sup>

for  $x \neq 2$  and  $x$  close to 2

$$\frac{1}{x-2} - \frac{4}{x^2-4} = \frac{1}{x-2} - \frac{4}{(x+2)(x-2)}$$

$$= \frac{x+2 - 4}{(x+2)(x-2)} = \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}} = \frac{1}{x+2}$$

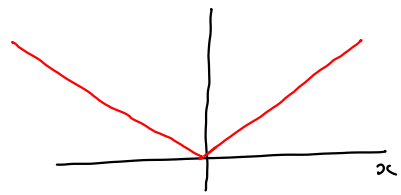
So

$$\lim_{x \rightarrow 2} \left( \frac{1}{x-2} - \frac{4}{x^2-4} \right) = \lim_{x \rightarrow 2} \frac{1}{x+2} = \frac{1}{4}$$

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Recall  $|-3| = 3$   
 $|3| = 3$

$$f(x) = |x|$$



Example

$$\lim_{x \rightarrow 0} \frac{x}{|x-1| - |x+1|}$$

Sol<sup>n</sup> For  $x \neq 0$  and  $x$  sufficiently close to 0

$$\frac{x}{|x-1| - |x+1|} = \frac{x}{1-x - (x+1)}$$

$$= \frac{\cancel{x}}{-2\cancel{x}} = -\frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{x}{|x-1| - |x+1|} = \lim_{x \rightarrow 0} -\frac{1}{2} = -\frac{1}{2}$$

## Weierstrass definition of a limit

We write

$$\lim_{x \rightarrow 1} g(x) = 6$$

to mean that:

for any real number  $\varepsilon > 0$   
there exists a number  $\delta > 0$   
such that

$$0 < |x - 1| < \delta$$

implies

$$|g(x) - 6| < \varepsilon .$$