Functions

A function $f: D \longrightarrow C$ consists of

$\mathbb{R}$ real numbers

1) a set D called a domain

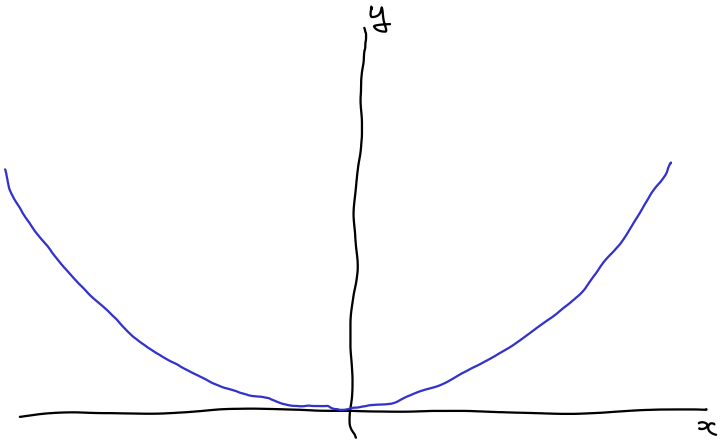
2) a set C called a codomain

3) a rule that assigns precisely one element $f(x) \in C$ to each input $x \in D$.

Example $f: \mathbb{R} \longrightarrow \mathbb{R}$, with $f(x) = x^2$

$D = \mathbb{R}$, $C = \mathbb{R}$

its graph is



The range of a function $f: D \to C$ is the set

$$\text{Range}(f) = \{ y \in \mathbb{R} : y = f(x) \text{ with } x \in D \}$$

Example (Continued)

$f: \mathbb{R} \to \mathbb{R}$, $x \longmapsto x^2$

$$\text{Range}(f) = \{ x \in \mathbb{R} : x \geq 0 \} =: \mathbb{R}^{\geq 0}$$

Convention

Often we use a formula such as

$$h(x) = \frac{x+2}{x-2}$$

to describe a function, without explicity stating the domain and codomain.

In this situation we always assume that the codomain is $\mathbb{R}$. We also assume that the domain is the largest possible subset of $\mathbb{R}$ for which the formula makes sense.

Example

1) $h(x) = \dfrac{x+2}{x-3}$

$D = \mathbb{R} \setminus \{3\}$

$C = \mathbb{R}$

Range $(h) = X = \mathbb{R} \setminus \{1\}$

2) $q(x) = \sqrt{x}$ positive square root of x

$D = \{x \in \mathbb{R} : x \geq 0\}$

$= [0, \infty)$

$C = \mathbb{R}$

Range $= [0, \infty)$

3) $p(x) = \sqrt{x^2 - 1}$

$C = \mathbb{R}$

$D = \{x \in \mathbb{R} : x \leq -1\} \cup \{x \in \mathbb{R} : x \geq 1\}$

$= (-\infty, -1] \cup [1, \infty)$

$\text{Range}(p) = [0, \infty) =: \mathbb{R}^{\geq 0}$

4) $r(x) = \sqrt{\frac{1}{x}}$

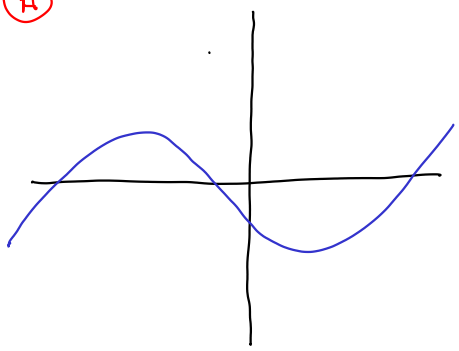
$C = \mathbb{R}$

$D = \{x \in \mathbb{R} : x > 0\} =: (0, \infty)$

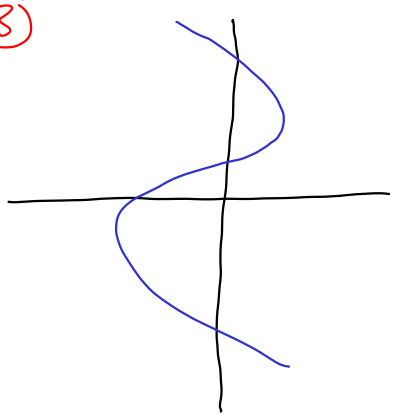
$\text{Range}(r) = \{x \in \mathbb{R} : x > 0\} =: (0, \infty)$

Question Which of the following are graphs of functions

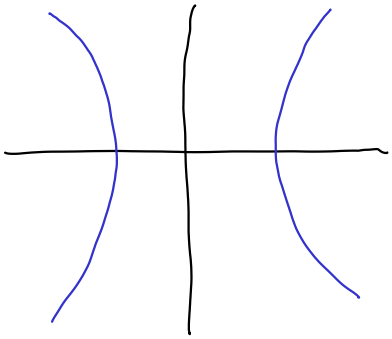
(A)



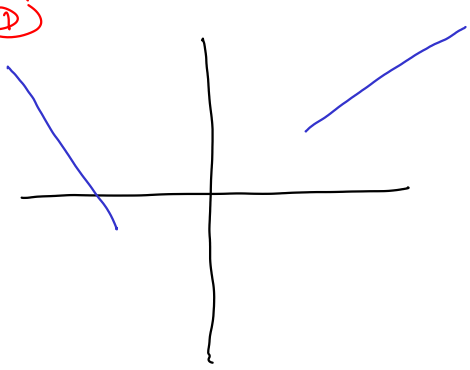
(B)



(C)



(D)



(B) is not a function

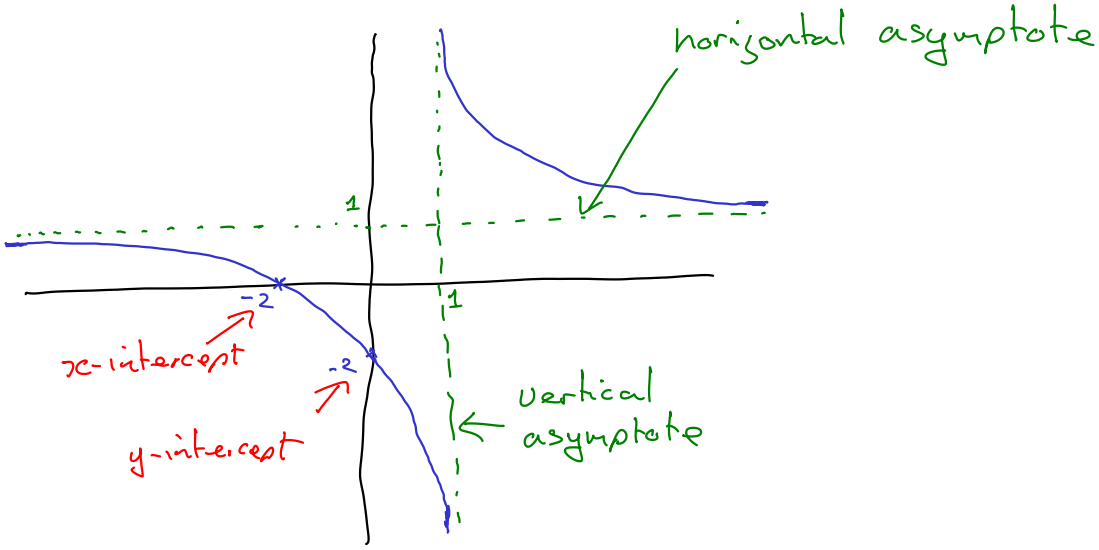
(C) is not a function

(A) is a function

(D) is a function, but its domain is a proper subset of $\mathbb{R}$.

Example $g : \mathbb{R}\setminus\{1\} \longrightarrow \mathbb{R}$ with $g(x) = \frac{x+2}{x-1}$

$D = \mathbb{R}\setminus\{1\}$, $C = \mathbb{R}$



Range $(g) = \mathbb{R}\setminus\{1\}$