MA313 Linear Algebra

Lecture notes and the problem sheet are available online:

http://www.maths.nuigalway.ie/~mstudies/ma313/

Overview

- **Lecturer:**
  Tobias Rossmann (Email: tobias.rossmann@nuigalway.ie)
  Office hours: Monday 3–4pm, Tuesday 2–3pm
- **Lectures:**
  Tuesday 1–2pm in AC202
  Friday 12 noon–1pm in AC214
- **Tutorials** (weeks 2–12)**:**
  Thursday 12 noon–1pm in IT206
- **Book:**
  D. C. Lay: "Linear Algebra and Its Applications", 4th ed.
- **Assessment:**
  - 50% final exam: 2 hour written exam
  - 30% continuous assessment. 3 in-class tests
  - 20% communication skills (jointly with MA335)
    More about this later!

Module content

We'll study

- vector spaces,
- linear transformations, and
- orthogonality,

and we'll see how these topics can be applied to

- signal processing,
- computer graphics, and
- data fitting.

Recap: previous modules on linear algebra

MA133

$\mathbb{R}^2$ and 2×2 matrices:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

MA203

$\mathbb{R}^n$, determinants, eigenvalues, row reduction, ...

What's left?

"Definition"

Informally, a **vector space** is a collection of *"things"* (which we call **vectors**) which can be *"manipulated like"* vectors in some $\mathbb{R}^n$ ... but without coming with intrinsic *"coordinates"*.

Why bother?

- Important examples of vector spaces don't come with "natural" coordinates.

  Example:
  function spaces which are e.g. used to represent signals
- Abstract vector spaces cast new light on $\mathbb{R}^n$.

  Example: what happens if you change coordinates?
- The theory of vector spaces illustrates the **development of mathematical theories** and the role of **abstraction** in mathematics.

Getting formal

Definition

A **vector spaces** consists of

- a (non-empty!) set V whose elements we call **vectors**,
- an operation called **addition** which assigns a vector

  $$u + v$$

  to any two vectors $u, v \in V$,
- an operation called **scalar multiplication** which assigns a vector

  $$cu$$

  to each scalar $c \in \mathbb{R}$ and vector $u \in V$

such that the axioms on the following slides are satisfied.

Getting formal

Definition (cont.)

We require that the following conditions **V1–V8** are satisfied for all vectors $u, v, w \in V$ and scalars $c, d \in \mathbb{R}$:

V1. $u + v = v + u$ (**commutativity** of addition)

V2. $(u + v) + w = u + (v + w)$ (**associativity** of addition)

V3. There exists $0 \in V$ called the **zero vector** such that

$$u + 0 = u$$

for all $u \in V$.

V4. For each $u \in V$, there exists $-u \in V$ such that $u + (-u) = 0$.

Getting formal

Definition (cont.)

V5. $c(u+v) = cu + cv$ (**distributivity I**)

V6. $(c+d)u = cu + du$ (**distributivity II**)

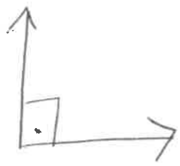
V7. $c(du) = (cd)u$

V8. $1u = u.$

Vector spaces:

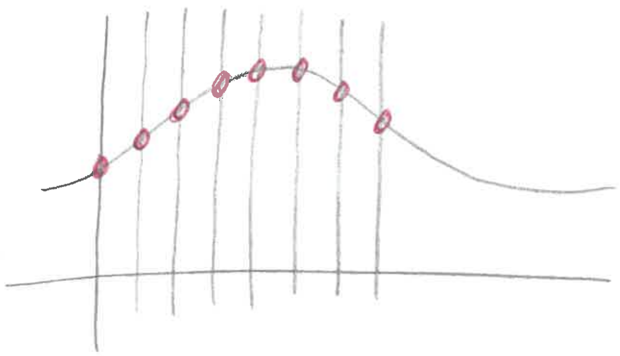
Linear transformations:

Orthogonality:



Applications:

- Signal processing



real-life signal (e.g. audio)

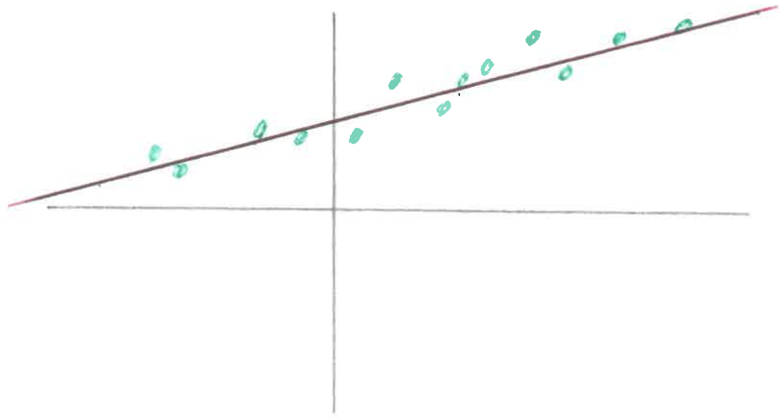
$\downarrow$ "quantisation"

"discrete signal"

$\downarrow$ "filter" (e.g. equaliser)

output

- Computer graphics (e.g. movement and projection).
- Data fitting: match "noisy" imprecise real-world data and an exact mathematical model



Example:

$\mathbb{R}^n = \left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} : x_1, \dots, x_n \in \mathbb{R} \right\}$ with addition

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} := \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

and scalar multiplication

$$c \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} := \begin{bmatrix} c x_1 \\ \vdots \\ c x_n \end{bmatrix}$$

is a vector space. The proof is tedious but easy!

E.g.

(V1) $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix} = \begin{bmatrix} y_1 + x_1 \\ \vdots \\ y_n + x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} + \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

Defn of +

addition in $\mathbb{R}$ is commutative

Defn of +, again

(V3) The zero vector is

$$0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}.$$

(V4) $-\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} -x_1 \\ \vdots \\ -x_n \end{bmatrix}$ works because

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} -x_1 \\ \vdots \\ -x_n \end{bmatrix} = \begin{bmatrix} x_1-x_1 \\ \vdots \\ x_n-x_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = 0,$$ the zero vector.

etc.