(b) $f'(0)$?

Using $f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$

$\Rightarrow f'(z) = [y] + i\,[2y]$

$\Rightarrow f'(0+0i) = 0 + 0i \quad (= 0).$

Lec $\boxed{7}$

Laplace's equation

If $f(z) = u(x,y) + i\, v(x,y)$ is analytic in a domain D, then u and v satisfy Laplace's equation

i.e. $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ①

and $\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ ②

[Notation,

we write $\frac{\partial u}{\partial x} = u_x$, $\frac{\partial^2 u}{\partial y^2} = u_{yy}$ etc.

Proof of ① Differentiating:

$u_x = v_y$ with respect to x

$u_y = -v_x$ with respect to y

we get:

$u_{xx} = v_{yx}$

and $u_{yy} = -v_{xy}$, respectively.

Adding the above two equations

we get $u_{xx} + u_{yy} = 0$

Proof of ② Differentiating:

$u_x = v_y$ with respect to y

$u_y = -v_x$ with respect to x

we get:

$u_{xy} = v_{yy}$

$u_{xy} = -v_{xx}$

Subtracting the above equations we get

$$0 = V_{yy} + V_{xx}.$$

i.e. $V_{xx} + V_{yy} = 0$

Any function that satisfies Laplace's equation is said to be a harmonic function

Note: we proved that if

$f(z) = u(x,y) + i\,v(x,y)$

is analytic,

then $u(x,y)$ and $v(x,y)$ are harmonic functions.

Ex

Show that

$u(x,y) = 2x + y + e^{x}\sin y$ is harmonic sol

$$u_x = 2 + e^x \sin y$$
$$u_{xx} = e^x \sin y$$

$$u_y = 1 + e^x \cos y$$
$$u_{yy} = -e^x \sin y$$

$$u_{xx} + u_{yy} = 0 \checkmark$$

Lec $\boxed{8}$

- If two harmonic functions $u(x,y)$ and $v(x,y)$ satisfy the C-R equations in a domain D, they are the real and imaginary parts of an analytic function F in D.

$[F(z) = u(x,y) + i\, v(x,y)]$

- $v(x,y)$ is said to be the conjugate harmonic function of u in D