Lec 4

Note 2:

Just like real functions, the above function is not defined when the denominator equals zero.

In this case, the values of z when

$$(z^2 - 2iz)^2 = 0$$

or $z^2 - 2iz = 0$

or $z(z - 2i) = 0$

or when $z = 0$ $(0+0i)$ And $z = 2i$ $(0+2i)$

Note 3

It is possible to find the derivative of these functions away from points when the denominator equals zero.

The same differentiation rules that apply to real functions apply to functions of a complex variable.

Ex

Differentiate $f(z) = \dfrac{ie^{iz}}{(z^2 - 2iz)^2}$

This is a quotient

Recall $d\left(\frac{u}{v}\right) = \dfrac{vu' - uv'}{v^2}$

$$\frac{du}{dz} = i\left[ie^{iz}\right] = i^2 e^{iz} = -e^{iz}$$

$$\frac{dv}{dz} = 2(z^2 - 2iz)(2z - 2i)$$

So

$$\frac{df(z)}{dz} = \frac{v\frac{du}{dz} - u\frac{dv}{dz}}{v^2}$$

with $\quad u = ie^{iz} \quad v = (z^2 - 2iz)^2$

$$f'(z) = \frac{(z^2-2iz)^2(-e^{iz}) - ie^{iz}\left[2(z^2-2iz)(2z-2i)\right]}{(z^2-2iz)^4}$$

$$= \frac{\left[-(z^2-2iz) - 2i(2z-2i)\right]e^{iz}}{(z^2-2iz)^3}$$

$$= -\frac{\left[(z^2-2iz) + 2i(2z-2i)\right]e^{iz}}{(z^2-2iz)^3}$$

$$= -\frac{\left[z^2 - 2iz + 4iz - 4i^2\right]e^{iz}}{(z^2-2iz)^3}$$

$$= -\frac{\left[z^2 + 2iz + 4\right]e^{iz}}{(z^2-2iz)^3}$$

Ex Given $f(z) =$

$$f(z) = \cos(z^2) + (z^3 + 2iz)^3$$

(a) find the finite z if any where $f(z)$ is not defined

(b) differentiate $f(z)$.

(a) $f(z)$ is defined at all finite z.

(There are no $\frac{1}{0}$ scenarios).

(b) $$\frac{df}{dz} = \left[-\sin(z^2)\right] \cdot (2z) + 3(z^3 + 2iz)^2 (3z^2 + 2i)$$

$$= -2z\sin(z^2) + 3(z^3 + 2iz)^2 (3z^2 + 2i)$$