$$\oint_{|z|=5} \frac{\sin z}{z^4}\, dz = \left(\frac{-1}{6}\right)(2\pi i)$$
$$= -\frac{\pi}{3} i$$

Lec 23

In the last example we found the <u>FULL</u> Laurent Series

but we only needed ONE coefficient i.e. of $\frac{1}{(z-z_0)^1}$ (b_1

If the pole has order 1

$$f(z) = \frac{b_1}{z-z_0} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \cdots$$

$$\Rightarrow \quad f(z)(z-z_0) = b_1 + a_0(z-z_0) + a_1(z-z_0)^2 + \cdots$$

$$\Rightarrow \quad \boxed{\lim_{z\to z_0}\left[f(z)(z-z_0)\right] = b_1}$$

Ex Find the residue (i.e. b_1) of $f(z) = \frac{e^z}{z}$ at $z = 0$

If the pole is order 1, then

$$b_1 = \lim_{z \to 0} f(z)(z - 0)$$

$$= \lim_{z \to 0} \frac{e^z}{z} \cdot z = \lim_{z \to 0} e^z = 1.$$

(check this by writing out the full Laurent series).

If the pole has order 2

$$f(z) = \frac{b_2}{(z-z_0)^2} + \frac{b_1}{z-z_0} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \cdots$$

$$\Rightarrow f(z)(z-z_0)^2 = b_2 + b_1(z-z_0) + a_0(z-z_0)^2 + \cdots$$

$$\left[f(z)(z-z_0)^2\right]' = b_1 + 2a_0(z-z_0) + 3a_1(z-z_0)^2 + \cdots$$

$$\boxed{\lim_{z \to z_0} \left[f(z)(z-z_0)^2 \right]' = b_1}$$

<u>Ex</u>

$$f(z) = \frac{50z}{(z+4)(z-1)^2}$$

Find: $\underset{z=1}{\mathrm{Res}}\ f(z)$

$$\lim_{z \to 1} \left[f(z)(z-1)^2 \right]'$$

$$= \lim_{z \to 1} \left[\frac{50z}{(z+4)(z-1)^2} \cdot (z-1)^2 \right]'$$

$$= \lim_{z \to 1} \frac{d}{dz} \left(\frac{50z}{z+4} \right)$$

$$= \left. \frac{(z+4)(50) - 50z}{(z+4)^2} \right|_{z=1}$$

$$= \frac{(5)(50) - 50}{25} = 8$$