So $f(z) = 1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \cdots$

$$= \sum_{n=0}^{\infty} \frac{2^n}{z^n}$$

Lec 21

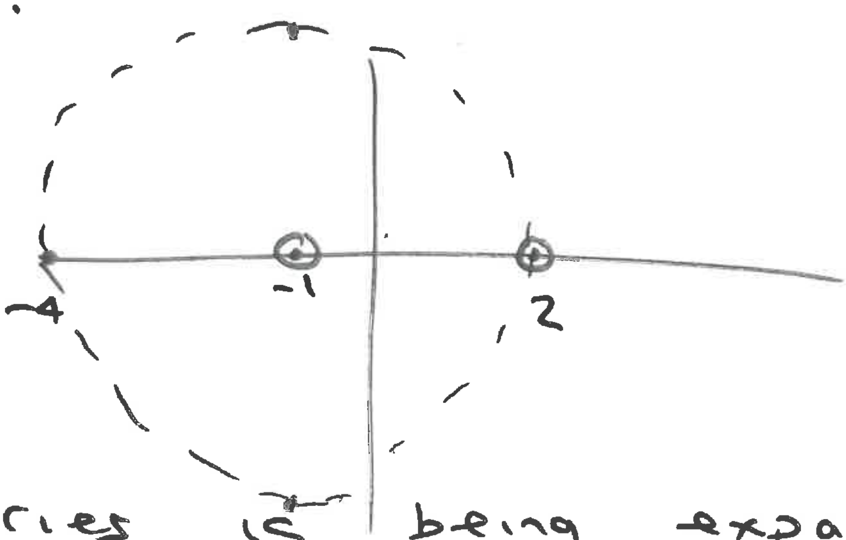
Ex Expand $\frac{3}{(z-2)(z+1)} = f(z)$

in a Laurent series valid

in (a) $0 < |z+1| < 3$

(b) $|z+1| > 3$.

Sol First note $f(z)$ is singular at $z = 2$ and $z = -1$.



The series is being expanded away away from these two singular points

Note also

you need to expand $f(z)$

around $z=-1$

i.e. in powers of $z+1$.

(a)

$$f(z) = \frac{3}{(z-2)(z+1)}$$

$$= \frac{3}{z+1}\left(\frac{1}{z-2}\right)$$

$$= \frac{3}{z+1}\left[\frac{1}{(z+1)-3}\right]$$

$$= -\frac{3}{z+1}\left[\frac{1}{3-(z+1)}\right]$$

$$= -\frac{3}{z+1}\frac{1}{3}\left[\frac{1}{1-\left(\frac{z+1}{3}\right)}\right] \qquad \text{Note: } \frac{1}{1-*}$$

$$= \frac{-1}{z+1}\left[1+\left(\frac{z+1}{3}\right)+\left(\frac{z+1}{3}\right)^2+\cdots\right]$$

when $\left|\frac{z+1}{3}\right|<1$

or $|z+1|<3$

as $z\neq-1$, we have $0<|z+1|<3$

So $f(z) = -\frac{1}{z+1} - \frac{1}{3} - \frac{z+1}{3^2} - \frac{(z+1)^2}{3^3} - \cdots$

(b) $f(z) = \frac{3}{(z-2)(z+1)}$

$$= \frac{3}{z+1} \frac{1}{z-2}$$

$$= \frac{3}{z+1} \frac{1}{(z+1)-3}$$

$$= \frac{3}{(z+1)^2} \left[\frac{1}{1 - \frac{3}{z+1}} \right] \quad \text{Note } \frac{1}{1-*}$$

$$= \frac{3}{(z+1)^2} \left[1 + \frac{3}{z+1} + \left(\frac{3}{z+1}\right)^2 + \cdots \right]$$

where $\left|\frac{3}{z+1}\right| < 1$

or $3 < |z+1|$

or $|z+1| > 3$

So $f(z) = \frac{3}{(z+1)^2} + \frac{3^2}{(z+1)^3} + \frac{3^3}{(z+1)^4} + \cdots$