$$z = x + iy \quad \text{—— (1)}$$
$$\bar{z} = x - iy \quad \text{—— (2)}$$

$\Rightarrow \quad z + \bar{z} = 2x \quad$ (by adding)

or $\quad x = \frac{1}{2}(z + \bar{z}) \quad (= Re(z))$

Subtracting (2) from (1), we get

$$z - \bar{z} = 2iy$$

or $\quad y = \frac{1}{2i}(z - \bar{z}) \quad (= Im(z))$

Lec
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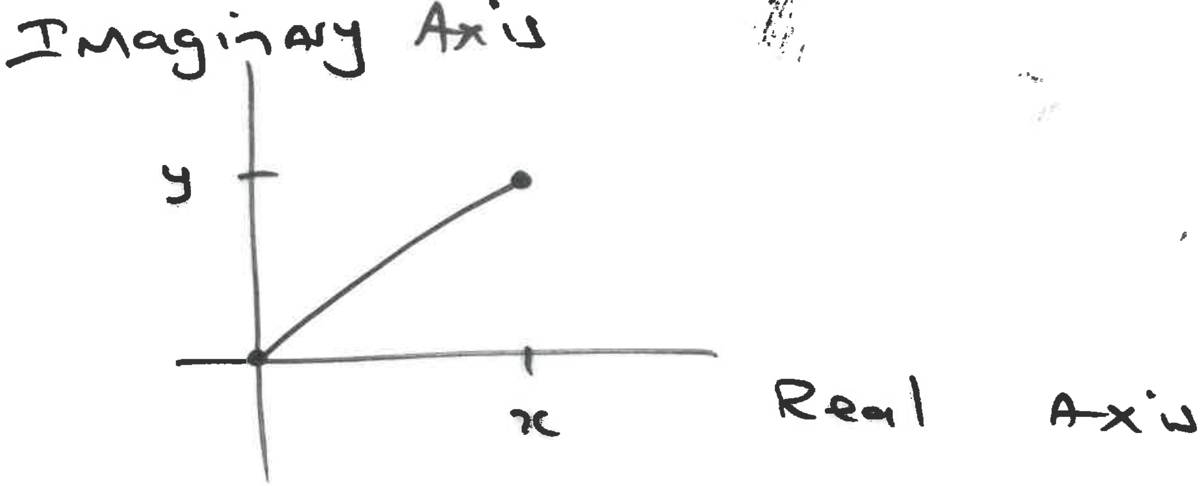
Exercise

If $\quad z_1 = x_1 + iy_1$
$\quad\quad z_2 = x_2 + iy_2 \quad$ Show:

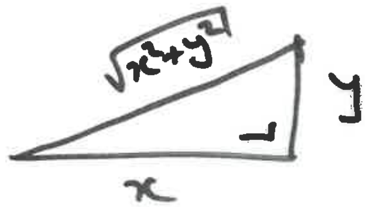
1. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
2. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
3. $\overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2$
4. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

Modulus of z or

(absolute value of z) denoted by $|z|$.



Recall:



So $|z| = \sqrt{x^2+y^2}$

or $|z| = \sqrt{z\bar{z}}$

because

$$z\bar{z} = (x+iy)(x-iy)$$
$$= x^2+y^2 + 0i$$
$$\Rightarrow \sqrt{z\bar{z}} = \sqrt{x^2+y^2} = |z|$$

Ex Find $|3+4i|$

$$|3+4i| = \sqrt{3^2+4^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5.$$

NB

It is possible to express any complex number expression in the form of

$x + iy \qquad x, y \in \mathbb{R}$.

Ex

Express $\dfrac{3-2i}{2+4i} + \dfrac{2-3i}{2-4i}$

in the form of $x + iy$

$x, y \in \mathbb{R}$.

$$\frac{(3-2i)(2-4i) + (2-3i)(2+4i)}{(2+4i)(2-4i)}$$

$$= \frac{(6-8) + [(3)(-4) + (-2)(2)]i + (4+12) + [(2)(4) + (-3)(2)]i}{4+16}$$

$$= \frac{-2 + [-12-4]i + 16 + [8-6]i}{20}$$

$$= \frac{14 - 14i}{20}$$

$$= \frac{7}{10} - \frac{7}{10} i$$

Lec

$\boxed{3}$

Functions of a Complex Variable.

In the same way that we have functions of a real variable, we can have functions of a complex variable.

As $z = x + iy$ $\quad x, y \in \mathbb{R}$

$i = \sqrt{-1}$

z involves the imaginary unit i