Lec 1

Complex Numbers / Complex plane.

Many equations have real solutions

e.g $x^2 = 4 \Rightarrow x = -2$ and $x = 2$.

Other equations do not have real solutions e.g

$$x^2 = -1$$

or $x^2 - 10x + 40 = 0$.

We introduce complex numbers to deal with the solutions of the above type of equations

A <u>complex number</u> z is an ordered pair (x, y) of real numbers x and y.

x is called the <u>real part</u>

we write $x = Re(z)$

y is the <u>imaginary part</u> of z.

We write $y = \mathrm{Im}(z)$.

Two complex numbers are equal if:

- their real parts are equal
- their imaginary parts are equal

"i" is called the imaginary unit.

$$i = (0, 1)$$

So if $z = (x, y)$

we can write $z = x + iy$

$x, y \in \mathbb{R}$

i is defined, such that

$$i^2 = -1$$

or $i = \sqrt{-1}$

Addition / subtraction

Let $z_1 = x_1 + i\,y_1$

$z_2 = x_2 + i\,y_2$

$z_1 + z_2 = (x_1 + x_2) + i\,(y_1 + y_2)$

$z_1 - z_2 = (x_1 - x_2) + i\,(y_1 - y_2)$

Multiplication

$z_1 . z_2 = (x_1 + i y_1)(x_2 + i y_2)$

$= x_1\,(x_2 + i\,y_2) + i y_1\,(x_2 + i\,y_2)$

$= x_1 x_2 + i\,x_1 y_2 + i\,y_1 x_2 + \underline{i^2}\,y_1 y_2$

As $i^2 = -1$

$z_1 . z_2 = (x_1 x_2 - y_1 y_2) + i\,(x_1 y_2 + x_2 y_1)$

<u>Ex</u> If $z_1 = 8 + 3i$

and $z_2 = 9 - 2i$

$Re(z_1) = 8$

$Im(z_2) = -2$

$z_1 + z_2 = (8+9) + (3-2)i$

$= 17 + i$

$$z_1 . z_2 = (8+3i).(9-2i)$$
$$= [8.9-(3)(-2)] + i[(8)(-2)+3(9)]$$
$$= [72+6] + i[-16+27]$$
$$= 78 + i(11)$$

Division.

Again let $z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$

To compute $\dfrac{z_1}{z_2}$ $(z_2 \neq 0)$

or $\dfrac{x_1 + iy_1}{x_2 + iy_2}$

or $\dfrac{x_1 + iy_1}{x_2 + iy_2} \times \dfrac{x_2 - iy_2}{x_2 - iy_2}$

$$= \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{(x_2^2 + y_2^2) + i(0)}$$

$$= \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}\right) + i\left(\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}\right)$$

Ex if $z_1 = 8+3i$
$z_2 = 9-2i$

$$\frac{z_1}{z_2} = \frac{8+3i}{9-2i}$$

$$= \frac{8+3i}{9-2i} \cdot \frac{9+2i}{9+2i}$$

$$= \frac{(72-6) + i(27+16)}{(81+4)}$$

$$= \frac{66}{85} + i\,\frac{43}{85}$$

Definition

The complex conjugate $\overline{z}$

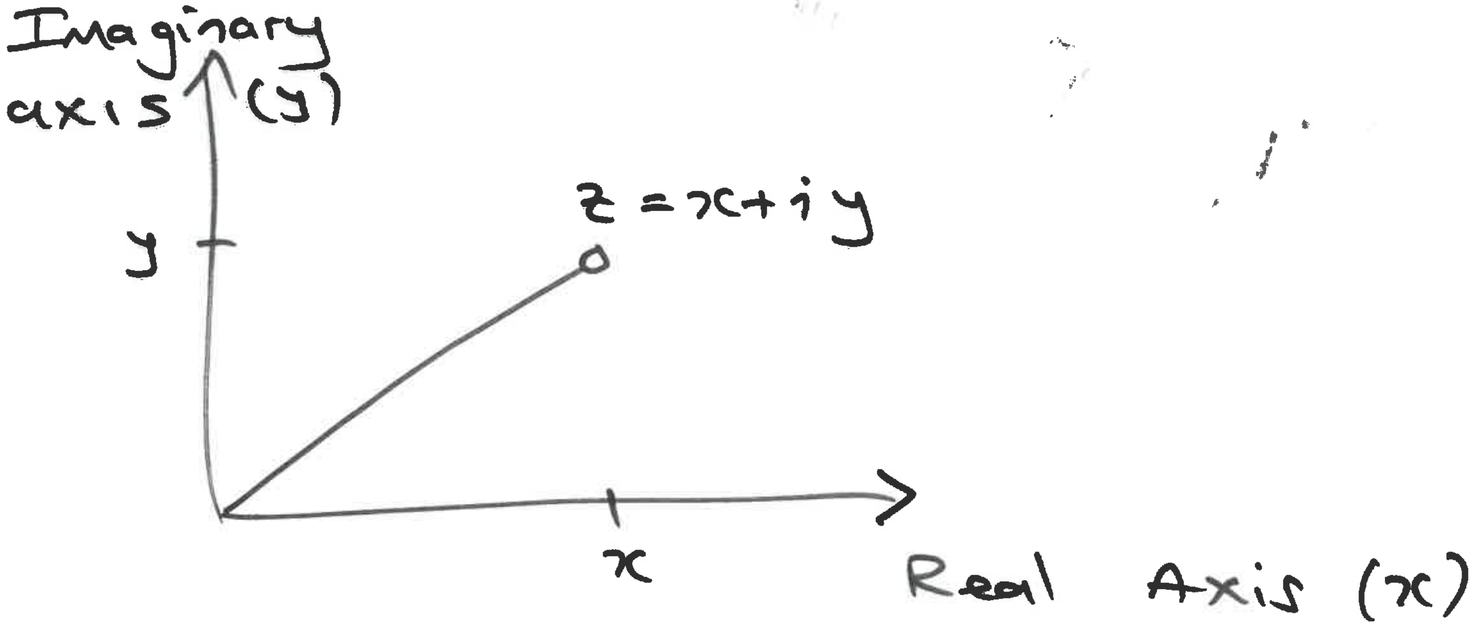
of a complex number

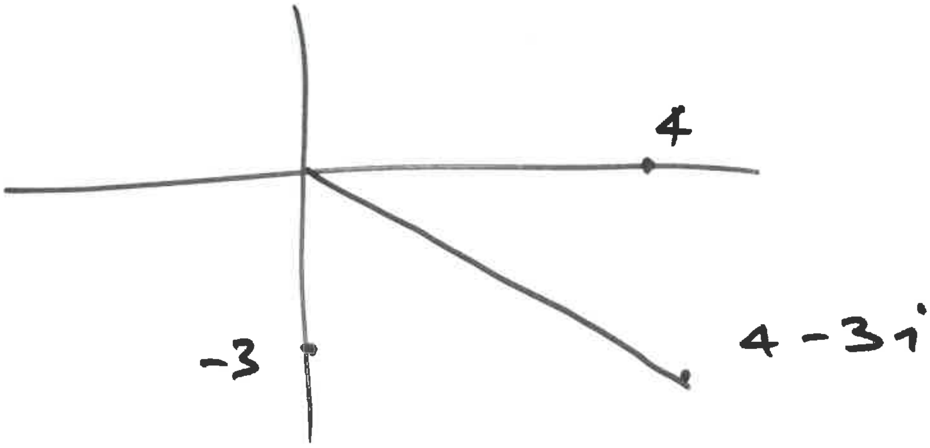
$$z = x + iy$$

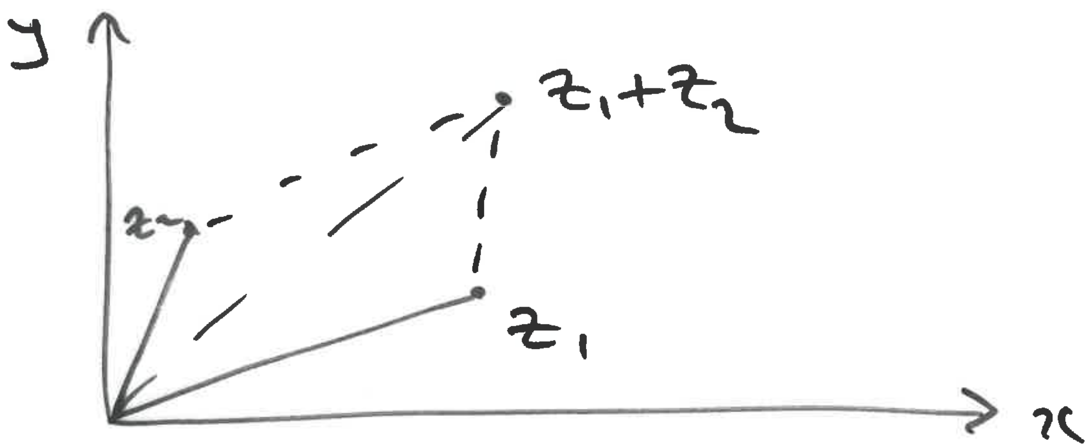
is defined to by

$$\overline{z} = x - iy.$$

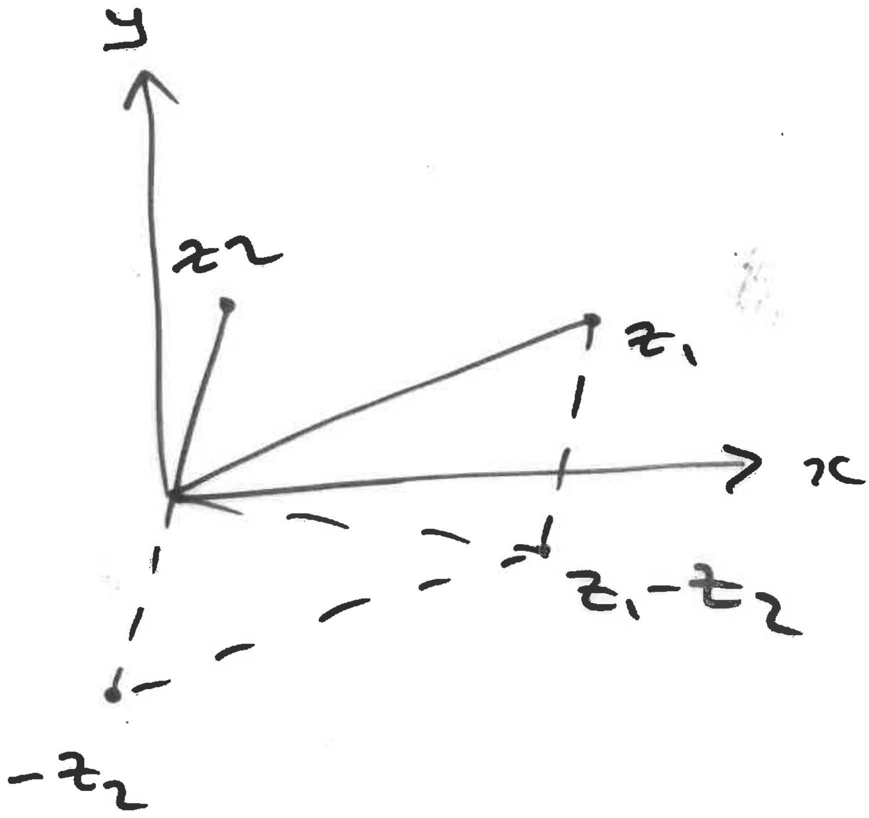
Complex Plane (Argand diagram).



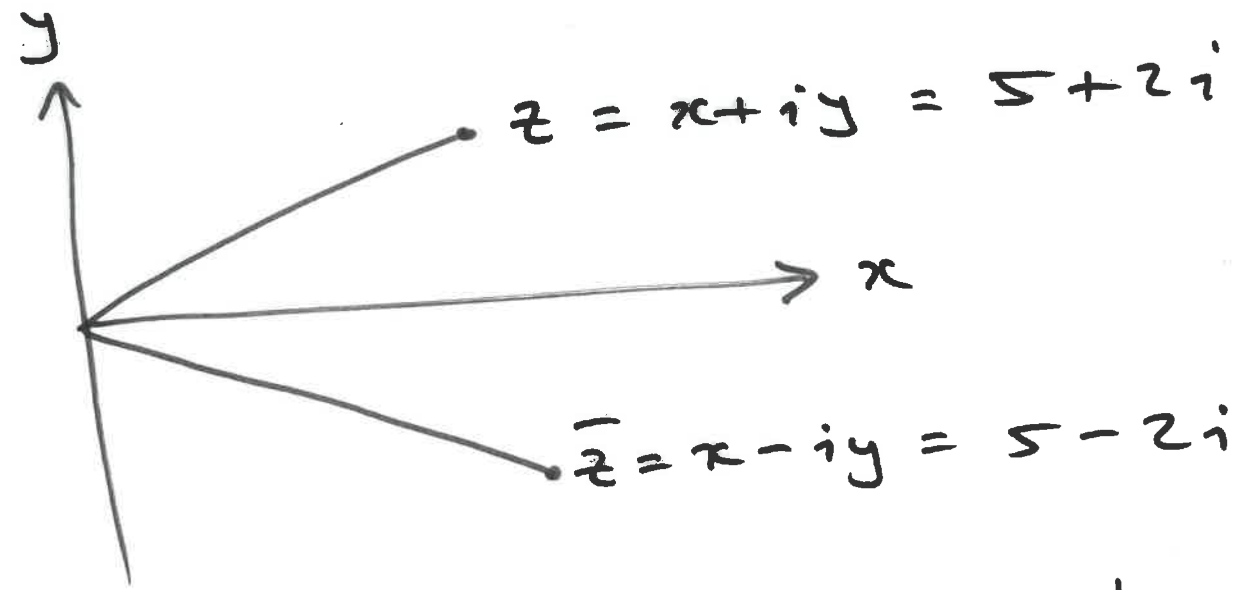




Addition of complex numbers



Subtraction of complex numbers.



i.e. to get the complex conjugate of z in the complex plane
→ reflect z in the real axis

$$z = x + iy \quad \text{—— (1)}$$
$$\bar{z} = x - iy \quad \text{—— (2)}$$

$\Rightarrow \quad z + \bar{z} = 2x \quad$ (by adding)

or $\quad x = \frac{1}{2}(z + \bar{z}) \quad (= \mathrm{Re}(z))$

Subtracting (2) from (1), we get

$$z - \bar{z} = 2iy$$

or $\quad y = \frac{1}{2i}(z - \bar{z}) \quad (= \mathrm{Im}(z))$

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Exercise

If $\quad z_1 = x_1 + iy_1$
$\quad\; z_2 = x_2 + iy_2 \quad$ Show:

1. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
2. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
3. $\overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2$
4. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$