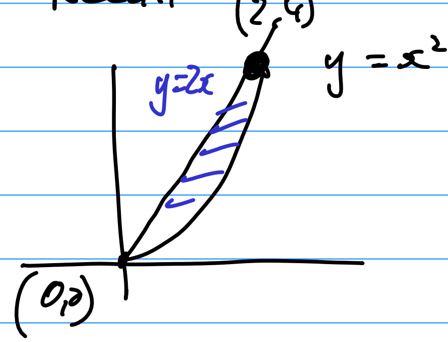
Lecture 7
W4L1

Type II region:

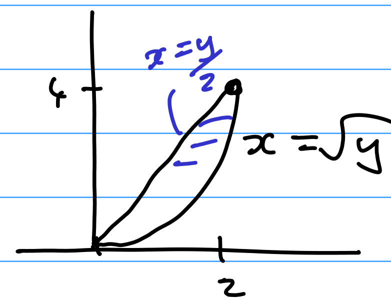
Compute $\displaystyle\int_0^4 \int_{y/2}^{\sqrt{y}} x^2+y^2 \, dx\, dy$

Recall



Also a Type I
with $y=2x$ on top
$y=x^2$ under

but also...



Type II with $x=\frac{y}{2}$ on left
$x=\sqrt{y}$ on right

Sol^n:

Inner: $\displaystyle\int_{x=y/2}^{\sqrt{y}} \left(x^2+y^2\right) dx$

$$= \left[\frac{x^3}{3} + y^2 x\right]_{x=y/2}^{\sqrt{y}}$$

$$= \left(\frac{y^{3/2}}{3} + y^{5/2}\right) - \left(\frac{y^3}{24} + \frac{y^3}{2}\right)$$

Outer: $\displaystyle\int_0^4 \frac{y^{3/2}}{3} + y^{5/2} - \frac{y^3}{24} - \frac{y^3}{2} \, dy$

$$= \left[\frac{1}{3} y^{5/2} \cdot \frac{2}{5} + y^{7/2} \cdot \frac{2}{7} - \frac{1}{24}\frac{y^4}{4} - \frac{1}{2}\frac{y^4}{4}\right]_0^4$$

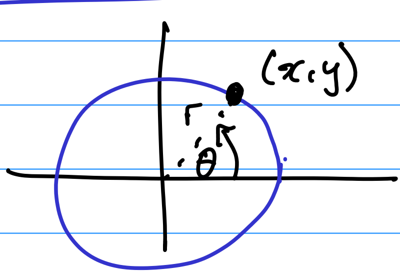
$$= \left(\frac{2}{15} \cdot 4^{5/2} + \frac{2}{7} \cdot 4^{7/2} - \frac{1}{96} \cdot 4^4 - \frac{1}{8} \cdot 4^4\right) - \left(0+0-0-0\right)$$

$$= \left(\frac{2}{15}(32) + \frac{2}{7}(128) - \frac{16}{6} - \frac{1}{2} \cdot 64\right)$$

$= \dots = \frac{216}{35} \approx 6.17$

as before.

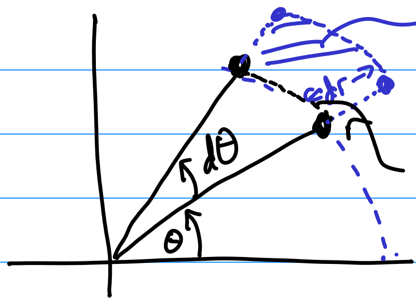
POLAR COORDINATES



$\rightarrow$ $x = r\cos\theta$
$y = r\sin\theta$

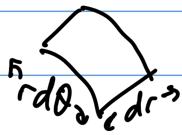
Cartesian form $\qquad$ polar form

"Polar rectangle"



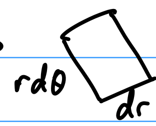
dA – increment in area of "polar rectangle" if r increases by dr & θ by $d\theta$

length of arc $= r\,d\theta$



$\longrightarrow$ becomes like a rectangle when $dr, d\theta \to 0$

thus $dA \longrightarrow$



$dA \sim r\,d\theta\,dr$

Say $\boxed{dA = r\,dr\,d\theta}$

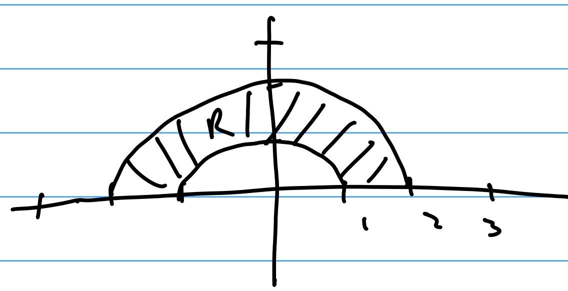
Ex (from Stewart) Compute $\iint_R (3x+4y^2)\,dA$

where R is the region in the upper half-plane bounded by

$x^2+y^2=1$ and

$x^2+y^2=4$

Sol^n: R is



R is this shaded annulus (half-annulus)

We describe R using natural polar coords

r ranges between 1 and 2

θ 0 and π

ie $1 \le r \le 2, \quad 0 \le \theta \le \pi$

Now, $\iint_R f(x,y)\,dA = \iint 3(r\cos\theta) + 4(r\sin\theta)^2 \, r\,dr\,d\theta$

where $x = r\cos\theta$, $y = r\sin\theta$

$$= \int_{r=1}^{2} \int_{\theta=0}^{\pi} \left(3r^2\cos\theta + 4r^3\sin^2\theta\right) d\theta\, dr$$

must decide whether to do θ or r first...