MA208 Quantitative Techniques for Business

Lecture 8: Probability ctd.

Dr Kirsten Pfeiffer

School of Mathematics, Applied Mathematics and Statistics
NUI Galway

Lecture 8 - Outline

Today we will talk about

- Random variables,
- Discrete / continuous random variables,
- Probability distributions,
- Binomial Probability Distribution.

Random Variables

A random variable is a numerical variable whose value is determined by an underlying random experiment.

Example

I toss a coin five times. This is a random experiment and the sample space can be written as

$$S = \{TTTTT, TTTTH, ..., HHHHH\}.$$

Note that here the sample space has $2^5 = 32$ elements. Suppose that in this experiment, we are interested in the number of heads. We can define a random variable X whose value is the number of observed heads. The value of X will be one of $0, 1, 2, 3, 4$ or 5 depending on the outcome of the random experiment.

Probability Distribution

A random variable has a probability distribution, which specifies the probability that its value falls in any given interval.

Example

Random variable	Probability
10 - 19	0.04
20 - 29	0.08
30 - 39	0.14
40 - 49	0.20
50 - 59	0.32
60 - 69	0.16
70 - 79	0.06

The random variable is the exam result received by a student.

Discrete / Continuous Random Variables

Depending on the type of the experiment, the numerical value that the random variable takes can be classified as discrete random variable or continuous random variable.

- **Discrete random variable**: the set of assumed values is *countable*, e.g. numbers of players in a team, number of visits in a doctor's surgery;
- **Continuous random variable**: the set of assumed values is *uncountable*, e.g. amount of rainfall in a month, the time required to cycle to university.

Examples

(1) Two balls are drawn in random in a succession without replacement from an urn containing four red balls and six black balls. Find the probabilities of all possible outcomes.

Solution

Let X denote the number of red balls in the outcome.

Possible outcomes	R R	R B	B R	B B
x_i	2	1	1	0

Here $x_1 = 2$, $x_2 = 1$, $x_3 = 1$, $x_4 = 0$

Discrete / Continuous Random Variables

Solution

The probability of getting 2 red balls when we draw the balls one at a time is

- Probability of 1st ball being red = $\frac{4}{10}$
- " " 2nd " " " = $\frac{3}{9}$

So $P(x_1) = \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

Similarly, we get

$$P(x_2) = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$$
$$P(x_3) = \frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$$
$$P(X_4) = \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$$

Check: If we have found all probabilities, they should add up to 1.

$$\frac{2}{15} + \frac{4}{15} + \frac{4}{15} + \frac{1}{3} = \frac{15}{15} = 1 \checkmark$$

Examples

(2) A jar coffee is picked at random from a filling process in which an automatic machine is filling coffee jars each with 1 kg of coffee. Due to some faults in the automatic process, the weight of a jar could vary from jar to jar between 0.9 kg and 1.05 kg. Let X denote the weight of a jar of coffee selected. What is the range of X?

Solution

range: $0.9 \leq x \leq 1.05$

(continuous variable)

Probability Distributions

There are some specific distributions that are used over and over in practice, thus they have been given special names. There is a random experiment behind each of these distributions. Since these random experiments model a lot of real life phenomenon, these special distributions are used frequently in different applications. That's why they have been given a name and we devote some time to study them.

Do not get intimidated by the complicated looking formulas, look at each distribution as a practice problem on discrete random variables.

Binomial Probability Distribution

Example

Suppose that I have a coin with $P(H) = p = 0.5$. I toss the coin $n = 30$ times and define X to be the total number of heads that I observe. Then X is binomial with parameters n and p. The range of X in this case is $\{0, 1, 2, ..., 30\}$. What is the probability to get 17 heads?

We can use the formula for Binomial Probability Distribution (see next slide):

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}.$$

Example

$$P(X = 17) = \binom{30}{17} \cdot (0.5)^{17} \cdot (1 - 0.5)^{(30-17)} \approx 0.111535052$$

Binomial Probability Distribution

The probability distribution of the random variable is called a binomial distribution if

- the experiments consist of a fixed number of n trials,
- the probability of success (denoted by p) remains constant,
- all trials are independent,
- each trial is either a success or failure (hence the name binomial).

The binomial distribution can be described by

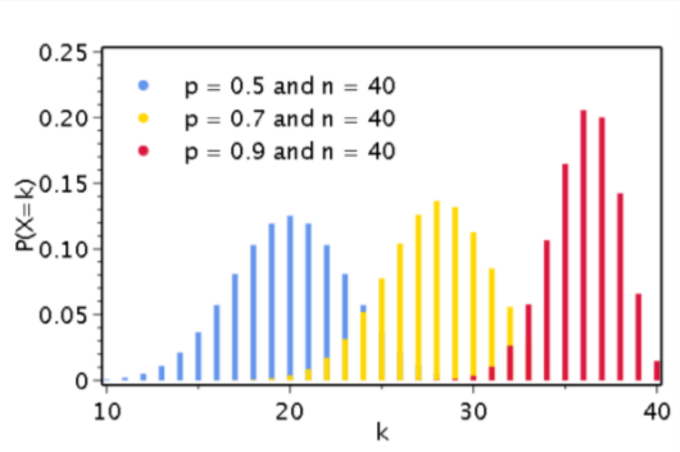
Binomial Probability Distribution

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where n is the number of trials, $x \in \{0, 1, 2, 3, ..., n\}$ is the discrete variable and p is the probability of success in a single trial.

Binomial Probability Distribution

Example



Binomial Probability Distribution

Example

Let 0.3 be the probability that a person will buy ice cream in a shop. Find the probability that among six people four will buy ice cream.

Solution

- independent events
- success / failure

} binomial distribution

Sln. $X :=$ # of persons who buy ice cream

$n = 6$ trials, we want 4 success

$$P(X=4) = \binom{6}{4} (0.3)^4 (1-0.3)^{(6-4)}$$

$$= \dots = 0.06$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Only 2 outcomes (getting a 5 or not), so binomial distn.

Sln. $n = 3$

Let x = number of fives appearing, $p = \frac{1}{6}$

(i) $x = 0$

$P(x=0) = \binom{3}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 = \frac{125}{216} = 0.5787$

(ii) $x = 1$

$P(x=1) = \binom{3}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 = \frac{75}{216} = 0.34722$

(iii) $x = 3$

$P(x=3) = \binom{3}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{1}{216} = 4.6296 \times 10^{-3}$

Binomial Probability Distribution

Example

A blindfolded marksman finds that on average he hits the target four times out of five. If he fires four shots, what is the probability of

(i) more than two hits?

(ii) at least three misses?

Solution

Again hit or miss → 2 possible outcomes → Binomial

Here $n = 4$, $p = \frac{4}{5} = 0.8$, X = number of hits

(a) $P(X > 2) = P(X = 3) + P(X = 4)$

$= \binom{4}{3}(0.8)^3(1-0.8)^1 + \binom{4}{4}(0.8)^4(0.2)^0 = \ldots = 0.8192$

(b) 3 misses ≙ 1 hit, 4 misses ≙ 0 hits. Looking for $P(X \leq 1)$

$P(X \leq 1) = P(X = 1) + P(X = 0) = \ldots = 0.0272$