MA208 Quantitative Techniques for Business

Lecture 22: Linear Programming ctd. - The Simplex Method

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Lecture 22

We learned how to formulate a linear programming problem, and how to solve such a problem in two variables.

Today we will discuss what to do if

- the problem requires an integer solution - and the optimum solution is not an integer, or
- the linear programming problem has more than two variables.

Consider the following problem from Lecture 20.

Example

Example

A distribution firm has to transport 1200 packages using large vans which can take 200 packages each and small vans which can take 80 packages each. The cost of running each large van is €40 and of each small van is €20. Not more than €300 is to be spent on the job. The number of large vans must not exceed the number of small vans.

(i) Formulate this problem as a linear programming problem given that the objective function is to *minimise* costs.

(ii) Solve the linear programming problem to calculate how many large and how many small vans the firm should send.

Example

Solution

(i) $x :=$ number of large vans
$y :=$ " " small vans

Minimise $C = 40x + 20y$,

subject to

$$200x + 80y \geq 1200$$
$$40x + 20y \leq 300$$
$$x \leq y$$
$$x \geq 0, \; y \geq 0$$

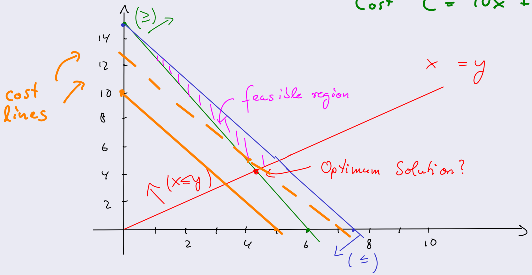
(ii) We sketch the feasible region and see that the optimum solution is not an integer solution.

Example

Solution (ctd.)

We need to identify the integer solutions within the feasible region and select the solution that intercepts the left-most cost line.

Cost $C = 40x + 20y \Rightarrow y = -2x + \frac{C}{20}$



$\Rightarrow$ Solution $x = 4$ and $y = 5$

So cost $= 40(4) + 20(5) = 260$

The Simplex Method

The corner point method shows that the optimum solution of a linear programming problem is always associated with a corner point of the solution space. The transition from the geometric corner-point solution to the simplex method entails a computational procedure that determines the corner points algebraically.
A main feature of the simplex method is that it solves the linear programming problem in iterations. Each iteration moves the solution to a new corner point that has the potential to improve the value of the objective function. The process ends when no further improvements can be realised.

To explain the method, let's look at an example.

The *Craic & Co.* Company Example

Craic & Co. produce both interior and exterior paints from two raw materials, $M1$ and $M2$. The following table provides the basic data of the problem:

	per ton of Exterior Paint	per ton of Interior Paint	Max. daily availability
Raw Material, $M1$	6 tons	4 tons	24 tons
Raw Material, $M2$	1 tons	2 tons	6 tons
Profit per ton (€1000)	5	4	

They also know that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. Also, the maximum daily demand of interior paint is 2 tons.
Craic & Co. want to determine the optimum (best) product mix of interior and exterior paints that maximises the total daily profit.

Formulating the Model

Let x be the tons produced daily of exterior paints and y be the tons produced daily of interior paints. Then we have the following constraints:

$$\begin{aligned} 6x + 4y &\leq 24 \\ x + 2y &\leq 6 \\ y - x &\leq 1 \\ y &\leq 2 \end{aligned}$$

as well as nonnegativity restrictions $x \geq 0$, $y \geq 0$.

The objective of the company is to maximise

$$p = 5x + 4y.$$