

# MA208 Quantitative Techniques for Business

## Lecture 21: Linear Programming - The Corner Point Method

Dr Kirsten Pfeiffer

School of Mathematics, Applied Mathematics and Statistics  
NUI Galway

## Lecture 21 - Linear Programming

Linear programming is a mathematical optimisation technique, where optimisation means maximizing or minimizing a linear equation subject to some constraints.

# Linear programming: the corner point method

## Example

A factory makes two types of machine, *A* and *B*. The table below details resource requirements for one of each type of machine.

Resource	Machine <i>A</i>	Machine <i>B</i>	Monthly availability
Energy	100 kWh	200 kWh	20,000 kWh
Steel	60 kg	80 kg	10,000 kg
Labour	2.5 h	2 h	400 h

The company makes a profit of €48 per machine *A* and €38 per machine *B* produced. How many of each type of machine should the company produce to maximize profit?

# Linear Programming

## Solution

Let  $x$  be the number of machine  $A$  and  $y$  be the number of machine  $B$ .

The total profit can be described by the **objective function**:

$$p = 48x + 38y$$

We must maximise this function subject to the following **constraints**:

$$100x + 200y \leq 20,000$$

$$60x + 80y \leq 10,000$$

$$2.5x + 2y \leq 400$$

We also assume that  $x \geq 0$  and  $y \geq 0$ .

# Linear Programming

## Corner Point Method

To solve a linear programming problem with two decision variables

- 1 Graph the region satisfying all of the inequalities. This is called **region of feasible solutions**.
- 2 Find the co-ordinates of all of the **corner points** of the region.
- 3 Substitute each into the function to be maximised.
- 4 One of these is the answer!

**Note:** To solve linear programming problems with more than two variables, more advanced methods are required, for example the “Simplex Method”.

# Linear Programming

## Solution (ctd.)

*Recall: we must*

*maximize*

$$48x + 38y$$

*subject to*

$$(1) \quad 100x + 200y \leq 20,000$$

$$(2) \quad 60x + 80y \leq 10,000$$

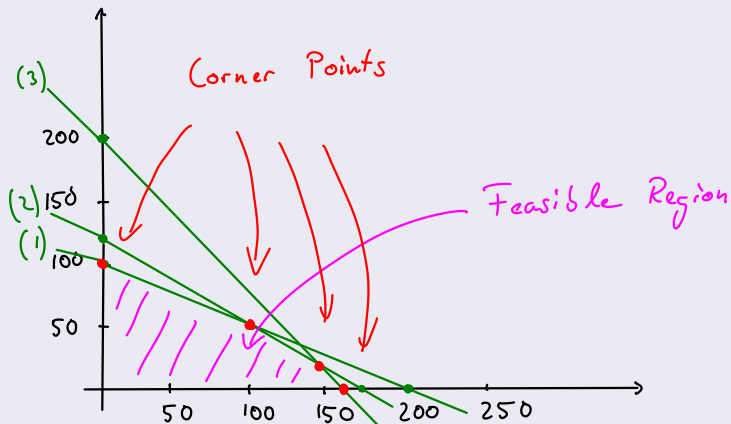
$$(3) \quad 2.5x + 2y \leq 400$$

*First, find the feasible region. To do that, draw graphs of the three inequalities.*

*The inequalities  $x \geq 0$  and  $y \geq 0$  indicate that the solution will lie in the first quadrant.*

# Linear Programming

## Solution (ctd.)



# Linear Programming

## Solution (ctd.)

*Now find the coordinates of the four corner points.*

**Fact:** *The optimal solution must occur at one of the corner points.*

*We know already two of the corner points, namely those on the  $x$ - and  $y$  axis:*

$$(0, 100) , (160, 0).$$

*The other two corner points are the intersections of lines (1) and (2) and the intersection of lines (2) and (3). Thus, we have to find those. (Board work!)*

# Linear Programming

## Solution (ctd.)

Find intersection of lines (1) and (2).

$$(1) \quad x + 2y = 200$$

$$\Rightarrow y = \frac{1}{2}(200 - x)$$

$$(2) \quad 6x + 8y = 1000$$

$$\Rightarrow y = \frac{1}{8}(1000 - 6x)$$

$$(y=y) \quad \frac{1}{2}(200 - x) = \frac{1}{8}(1000 - 6x)$$

$$4(200 - x) = 1000 - 6x$$

$$800 - 4x = 1000 - 6x$$

$$2x = 200$$

$$x = 100$$

$$x = 100 \Rightarrow y = \frac{1}{2}(200 - 100) = 50. \quad \text{So corner point is } (100, 50)$$

Similarly, calculate intersections of lines (2) and (3).

# Linear Programming

## Solution (ctd.)

*The intersection point of lines (1) and (2) is (100, 50),  
and the intersection point of lines (2) and (3) is (150, 12.5).*

*So the **corner points** are*

*(0, 100), (160, 0), (100, 50), (150, 12.5).*

*To find out which of those is the solution, we need to substitute  
each of them into the profit function*

$$p = 48x + 38y$$

# Linear Programming

## Solution (ctd.)

$$p = 48x + 38y$$

- $(0, 100)$        $p = 48(0) + 38(100) = 3,800$
- $(160, 0)$        $p = 48(160) + 38(0) = 7,680$
- $(100, 50)$        $p = 48(0) + 38(100) = 3,800$
- $(150, 12.5)$        $p = 48(0) + 38(100) = 3,800$

*So the maximum profit is €7,680 which happens when the factory makes 160 machines of type A and 0 machines of type B.*

## Example

### Example (from 2015 exam paper)

An oil company wish to inject a fracking fluid into the ground of an island and can use two types of chemicals, Bromine and DBNPA. Both types have an impact on the environment and therefore there are limits on the impact type.

<b>Impact type</b>	<b>Bromine</b>	<b>DBNPA</b>	<b>Limit for permitted damage</b>
Drinking water	40 kTpU	20 kTpU	1200 kilotons
Air pollution	30 kTpU	30 kTpU	1800 kilotons
Surface contamination	25 kTpU	50 kTpU	1500 kilotons

(kTpU = kilotons per unit)

# Example

## Example (ctd.)

- (a) Let  $x$  and  $y$  denote the number of units to be extracted with Bromine (resp. DBNPA). Write down three inequalities representing the constraints on  $x$  and  $y$  imposed by the above table.
- (b) Sketch these inequalities and shade the region of feasible solutions.
- (c) The company makes a profit of €13 million per unit of gas extracted with Bromine and of €7 million per unit extracted with DBNPA. How many units extracted with Bromine together with how many extracted with DBNPA do maximize the profits? And what is the maximum profit?

# Example

Solution

# Example

Solution (ctd.)

## Example

### Example

The annual subscription for a tennis club is €20 for adults and €8 for juniors. The club needs to raise at least €800 in subscriptions to cover its expenses.

The total number of members is restricted to 50. The number of junior members is to be between one quarter and one third of the number of adult members.

- (i) Represent this problem graphically.
- (ii) Find the number of adults and junior members which will bring the largest amount of money in subscriptions.