

MA208 Quantitative Techniques for Business

Lecture 14: Mathematics of Finance ctd.

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Lecture 14 - Outline

Today we will talk about

- Present value of an Annuity
- Amortization

Revision: Present value of an Annuity

Present value of an Annuity

In general the Present Value of an Ordinary Annuity is

$$PV = PMT \frac{(1 - (1 + i)^{-n})}{i}$$

where PV = present value of all payments, PMT = periodic payment, i = rate per period, and n = number of periods.

Note: Payments are made at the end of each period.

Present value of an Annuity

Example

What is the present value of an annuity that pays €200 per month for five years if money is worth 6% compounded monthly?

Solution

$$PMT = € 200, \quad i = \frac{0.06}{12} = 0.005, \quad n = (5)(12) = 60$$

$$\begin{aligned} PV &= PMT \frac{1 - (1+i)^{-n}}{i} \\ &= 200 \frac{1 - (1.005)^{-60}}{0.005} \\ &= € 10,345 \end{aligned}$$

Amortization

The present value formula for an ordinary annuity is also used to determine the distribution of loan repayments, as determined by an *amortization* schedule.

Example

Suppose you borrow €5000 from a bank to buy a car and agree to repay the loan in 36 equal monthly payments, including all interest due. If the bank charges 1% per month on the unpaid balance (i.e. 12% per year compounded monthly), how much should each payment be?

Amortization

Here, the bank has bought an annuity from you. So the question is: If the bank pays you $PV = €5000$ for an annuity, what are the periodic payments PMT ?

Solution

We have $PV = 5000$, $i = 0.01$, $n = 36$.

$$PV = PMT \frac{(1 - (1 + i)^{-n})}{i}$$
$$5000 = PMT \frac{(1 - (1.01)^{-36})}{0.01}$$
$$PMT = €166.07$$

Amortization

Example

You wish to purchase a retirement plan that earns **6.5%** interest compounded annually. You will make annual deposits into this account for **25** years, and then make **20** equal withdrawals of **€10,000** reducing the balance to zero. Compute

- 1 the value of the fund based on the withdrawals,
- 2 the amount of each deposit in order to maintain the fund,
- 3 the total interest earned over the entire 45 years.

Solution

Have a go at this question yourself!

Amortization

Solution

We will compare results on Monday.