Semester II Examinations 2017/2018

Exam Codes	1EM1, 1OA1, 2BA1, 2BCW1, 2BCT1, 2BPT1, 2BS1, 2EH1, 3BS9
Exam	Second Year Arts and Science Third Year Science
Module	LINEAR ALGEBRA
Module Code	MA203
External Examiner	Prof. Thomas Brady
Internal Examiners	Prof. Graham Ellis Dr. John Burns

Instructions **Answer all questions.**

Duration	2 hours
No. of Pages	4 pages including this page
School	Mathematics, Statistics & Applied Mathematics

Requirements:

Release in Exam venue	Yes	☑	No	☐
MCQ	Yes	☐	No	☑
Statistical / Log Tables	Yes	☑	No	☐

Q1. (a) [13 marks] Consider the following system of equations

$$\begin{array}{rcrcrcrcl} 2x_1 & - & x_2 & - & x_3 & + & x_4 & = & 0 \\ x_1 & - & x_2 & + & x_3 & - & x_4 & = & 3 \\ x_1 & & & + & x_3 & + & x_4 & = & 2 \end{array}$$

(i) Write down the augmented matrix for this system of equations.

(ii) Using elementary row operations, convert the augmented matrix to reduced row echelon form.

(iii) Write down the general solution of the system of equations.

(b) [12 marks] For each of the following statements, declare whether the statement is true or false.

(i) If a system of linear equations has more equations than unknowns then it has at least one solution.

(ii) $2x + 3y - 5z = 2$ is the equation of a plane through the origin in $\mathbb{R}^3$.

(iii) It is possible to determine if two lines in $\mathbb{R}^3$ intersect by solving an appropriate system of linear equations.

Q2. (a) [8 marks] Let

$$A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}.$$

Of the products AB, BA, A^2 and B^2, compute all those that are defined.

(b) [11 marks] Use elementary row operations to find the inverse of the matrix A below. Determine the rank of A and the kernel of A.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 4 & 1 & 5 \\ 4 & 1 & 4 \end{pmatrix}.$$

(c) [6 marks] For each of the following statements, declare whether the statement is true or false.

(i) Elementary row operations do not change the determinant of a matrix.

(ii) If a system of linear equations has a square coefficient matrix A with nonzero determinant, then it has a unique solution.

(iii) If a square matrix A is invertible, then the rows of A are linearly independent.

Q3. (a) [10 marks] Express the vector $(5, -1, 7)$ as a linear combination of the vectors $(1, -1, 0), (1, 0, 2)$ and $(0, 1, 1)$. What is the span of the set of vectors $\{(1, -1, 0), (1, 0, 2), (0, 1, 1)\}$?

(b) [6 marks] Let $n = (n_1, n_2, n_3) \in \mathbb{R}^3$ and consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $L(v) := n \times v$ (the cross product of n and v). Find the first column of the matrix for L (w.r.t. the standard basis for $\mathbb{R}^3$).

(c) [9 marks] For each of the following statements, declare whether the statement is true or false.

(i) If 0 is an eigenvalue of a square matrix A, then A has no inverse matrix.

(ii) If the characteristic polynomial of a 3×3 matrix A is $P(\lambda) = \lambda^3 - \lambda^2 + 4\lambda - 4$, then 1 is an eigenvalue of A.

(iii) A 2×2 matrix can have three distinct eigenvalues.

Q4. The town of Ballymarcove has three mobile phone providers ONE, TWO and THREE and every resident of Ballymarcove is a customer of exactly one provider.

- Every year 10% of ONE customers switch to TWO and 20% switch to THREE (with 70% remaining with ONE).
- Every year 30% of TWO customers switch to ONE and 20% switch to THREE (with 50% remaining with TWO).

- Every year 30% of THREE customers switch to ONE and 30% switch to TWO (with 40% remaining with THREE).

(a) [9 marks] Write down the transition matrix for this Markov process.

(b) [8 marks] Explain why the transition matrix has 1 as an eigenvalue.

(c) [8 marks] TWO is a relative newcomer to Ballymarcove and in the long term it aims to have a third of the market. Currently ONE has 60% of the market, TWO has 10% and THREE has 30%. If current trends continue, can TWO expect to achieve their aim?