So in general if $P=(P_1,P_2,P_3)$ & $v=(v_1,v_2,v_3)$ the eqn is $(t=)$ $\dfrac{x_1-P_1}{v_1} = \dfrac{x_2-P_2}{v_2} = \dfrac{x_3-P_3}{v_3}$

Question: (To be answered later by solving simultaneous linear equations.)

Let L_1: $\dfrac{x_1-3}{4} = \dfrac{x_2-1}{2} = \dfrac{x_3+1}{3}$

& L_2: $x_1-1 = \dfrac{x_2-2}{4} = \dfrac{x_3-1}{2}$.

Do L_1 and L_2 intersect, & if so find their intersection.

11
5
6

In parametric form we have

L_1: $(3,1,-1)+t(4,2,3)$ &

L_2: $(1,2,1)+s(1,4,2)$.

So where they meet, the x_1, x_2, & x_3 coordinates must be the same. i.e.,

$$\left.\begin{aligned} 3+4t &= 1+s \\ 1+2t &= 2+4s \\ -1+3t &= 1+2s \end{aligned}\right\} \text{OR} \left\{\begin{aligned} 4t-s &= -2 \\ 2t-4s &= 1 \\ 3t-2s &= 2 \end{aligned}\right.$$

We don't expect any sol'ns as geometrically, in general 2 lines in $\mathbb{R}^3$ won't meet.

Similarly, too many eq'ns i.e, 3 eqns and only 2 unknowns.

<u>The dot product</u> (encodes length and angle between vectors)

Def^n: Let $u = (u_1, u_2, \dots, u_n)$ & $v = (v_1, v_2, \dots, v_n)$. Then the dot or scalar product of u & v is the real number denoted by $u \cdot v$ and defined by

$$u \cdot v = u_1 v_1 + u_2 v_2 + \dots + u_n v_n$$
$$= \sum_{i=1}^{n} u_i v_i$$

Ex, $u = (1, 2, 1)$ $\quad v = (-1, 2, 2)$

$u \cdot v = (1)(-1) + (2)(2) + (1)(2) = 5.$

$v \cdot u = u \cdot v = 5.$

Ex) $u = (2,3)$ $\quad v = (-3,2)$ then

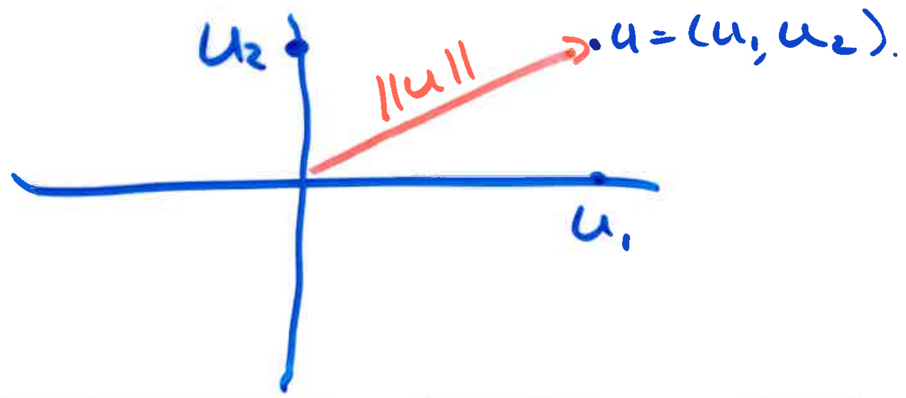
$$u \cdot v = -6 + 6 = 0$$
$$u \cdot u = 2^2 + 3^2 = 13$$

$u \cdot u$ is the distance squared from $u = (2,3)$ to $(0,0)$, i.e., the length $\|u\|$ of the vector u squared. (In example above, $\|(2,3)\| = \sqrt{13}$.)

In general by Pythagoras, we have:

- If $u = (u_1, u_2) \in \mathbb{R}^2$ then

$$\|u\|^2 = u_1^2 + u_2^2 \Rightarrow \|u\| = \sqrt{u_1^2 + u_2^2}$$



Def^n: Let $v = (v_1, \dots, v_n) \in \mathbb{R}^n$. The <u>norm</u> of v, or length of v, denoted $\|v\|$ is defined to be

$$\|v\| = \sqrt{v \cdot v} = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}.$$

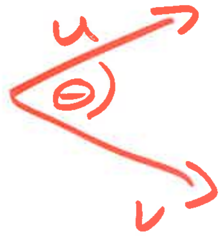
Properties of the dot product:

(i) $u \cdot v = v \cdot u \quad \forall\, u, v \in \mathbb{R}^n$

(ii) $u \cdot (v + w) = u \cdot v + u \cdot w \quad \forall\, u, v, w \in \mathbb{R}^n$

(iii) $(ru) \cdot v = u \cdot (rv) = r(u \cdot v) \quad \forall\, u, v \in \mathbb{R}^n,\ r \in \mathbb{R}$

(iv) In $\mathbb{R}^2$, $u \cdot v = \|u\|\,\|v\| \cos\theta$, where θ is the angle between u & v



As in higher dimensions, any 2 vectors define a 2-dimensional plane, we also have the same notion of angle θ.

Corollary: $\quad u \cdot v = 0 \iff \cos\theta = 0$

i.e., $\theta = \dfrac{\pi}{2}$

So u & v are perpendicular or orthogonal.

Def^n: The hyperplane in $\mathbb{R}^n$ with normal $n = (n_1, \dots, n_n)$ through the origin $O = (0, 0, \dots, 0)$ is the set of vectors (or points) $x = (x_1, \dots, x_n)$ that are perpendicular to n.