

MA203 - Linear Algebra - Class test 2

Attempt **ALL** questions - Total of 40 marks

1. Let $A = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 0 \\ 2 & -3 \end{pmatrix}$.

Determine if each of the matrix products AB , BA , A^2 and B^2 are defined or undefined, and calculate the product if possible. (10)

2. (a) Prove that the vectors $(1, 2, 1, 2)$, $(2, 1, 2, 1)$, $(1, 1, 0, 0)$, and $(0, 0, 1, 1)$ are not linearly independent i.e. are linearly dependent. (6)

(b) If A is the matrix with rows equal to the vectors of part (a), what is the rank of A ? What is the dimension of the kernel of A ? (4)

3. Use elementary row operations to find the inverse of the matrix (10)

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}.$$

4. Find the matrix (w.r.t. the standard basis for \mathbb{R}^3) of the linear map $L : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, where $L(x, y, z) = (2x, y, 0)$. (10)

BONUS: If S is a sphere in \mathbb{R}^3 , describe the appearance of $L(S)$, looking directly at the xy -plane. (5)

NOTE: A maximum grade of 40 marks is obtainable, i.e., up to 45 marks are awarded for the questions but totals between 40 and 45 will be recorded as 40.