Similarly the matrix for a rotation about the e_1 axis is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}$

Exercise: Verify this, and find the matrix for a rotation about the e_2 axis.

Ex: Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be projection of $\mathbb{R}^3$ onto the e_1-e_2 plane

i.e., $L(x_1, x_2, x_3) = (x_1, x_2)$

So $L((1,0,0)) = (1,0)$

$L((0,1,0)) = (0,1)$

$L((0,0,1)) = (0,0)$

$\Rightarrow A_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Exercises: For the examples of linear transformations we have seen, prove that they are in fact linear.

That is, show (i) $L(u+v) = L(u) + L(v)$ $\forall u, v$

& (ii) $L(kv) = kL(v)$ $\forall k \in \mathbb{R}$.

Revision of Matrix Multiplication

Recall that matrix multiplication was so defined so that if $L_1 \longleftrightarrow A$ & $L_2 \longleftrightarrow B$ then

$$L_1 \circ L_2 \longleftrightarrow AB \quad (\neq BA \text{ usually}).$$

So if $v = (x_1, x_2, \dots, x_n)$, to find

$$L_1 \circ L_2(v) = L_1(L_2(v)) \qquad (L_2 : \mathbb{R}^n \to \mathbb{R}^m,\ L_1 : \mathbb{R}^m \to \mathbb{R}^p)$$

we place v behind the matrix AB as a column and multiply.

$$\underset{p\times m}{A}\,\underset{m\times n}{B}\,\underset{n\times 1}{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}} := \begin{pmatrix} y_1 \\ \vdots \\ y_p \end{pmatrix}$$

$p \times 1$

and $L_1 \circ L_2(v) = (y_1, y_2, \dots, y_p)$

Recall: AB only makes sense if the number of entries in a row of A = the number of entries in a column of B.

i.e., if A is $p \times m$ and B is $m \times n$

then AB is $(p \times \underbrace{\cancel{m})\,(\cancel{m}}_{\text{equal}} \times n) = p \times n$.

If $R = r_1 \; r_2 \; \cdots \; r_m$ is a row of A

& $C = \begin{matrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{matrix}$ is a column of B

Then $RC := r_1 c_1 + r_2 c_2 + \cdots + r_m c_m$

and we obtain the entries of AB by multiplying all the rows of A by all the columns of B & placing the answer in the corresponding position of AB.

e.g., i^{th} row of A multiplied by the j^{th} column of B is placed in the i^{th} row and j^{th} column of AB.

Ex: $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$

$$2 \times \underbrace{3 \qquad\qquad 2}_{\neq} \times 2$$

So AB is undefined.

But BA $\quad (2 \times \underbrace{2) \times (2}_{=} \times 3)$ is a 2×3 matrix.

$$BA = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \end{pmatrix} = \begin{pmatrix} (2\,3)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (2\,3)\begin{pmatrix} 2 \\ -1 \end{pmatrix} & (2\,3)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ (1\,1)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (1\,1)\begin{pmatrix} 2 \\ -1 \end{pmatrix} & (1\,1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{pmatrix}$$