MA203 LINEAR ALGEBRA

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Lectures: Wed 13:00 Anderson
Fri 11:00 AM150

Tutorials: TBC.

Course text: "Linear Algebra & Its Applications"
David C. Lay.

Assessment:

*M. Studies: 40% CA 60% Exam.

Others: 30% CA 70% Exam.

CA = 3 class tests (+ essay*)

Introduction: Geometric Background
i.e., vectors & the dot product.

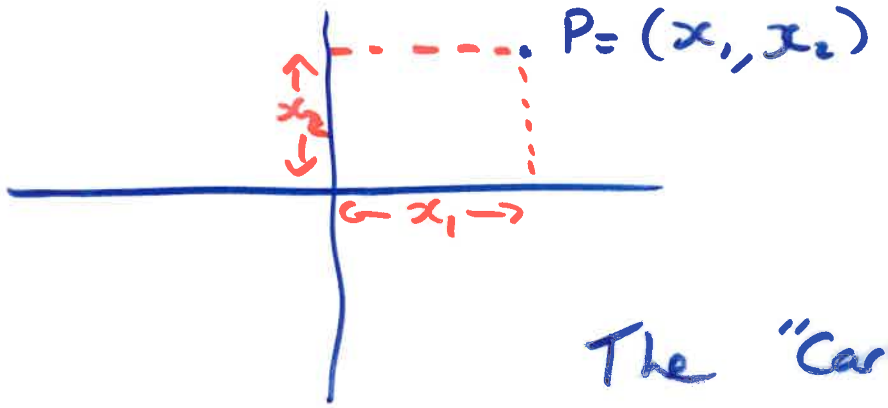
Recall: $\mathbb{R}$ denotes the set of all real numbers, and $\mathbb{N}$ denotes the set of all natural numbers. i.e., $\{1, 2, \ldots, \}$

Let $n \in \mathbb{N}$. Then $\mathbb{R}^n = \{(x_1, \ldots, x_n) \mid x_i \in \mathbb{R}\}$.

An element $v = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is called a <u>vector</u>.

Pictures: $n=1$: $\mathbb{R}$:

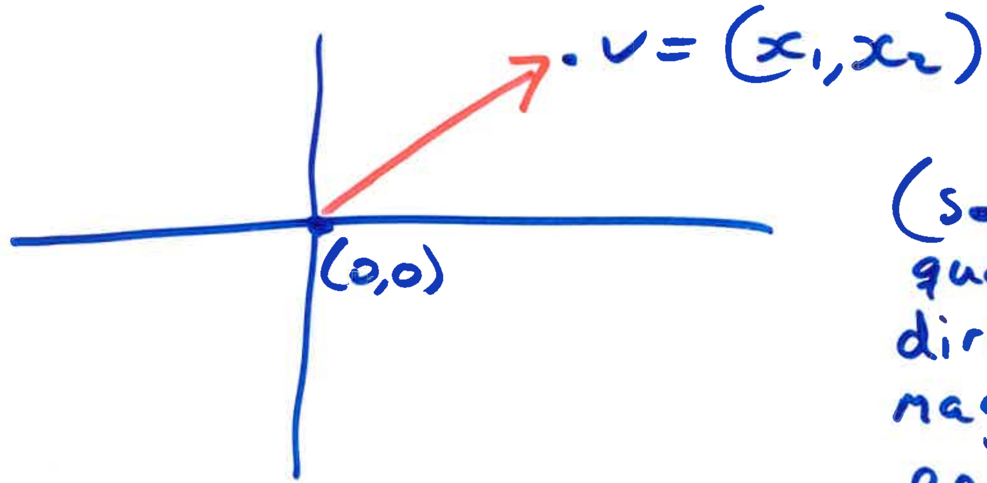
$n=2$: $\mathbb{R}^2 = \{(x_1, x_2) \mid x_1, x_2 \in \mathbb{R}\}$



The "Cartesian Plane"

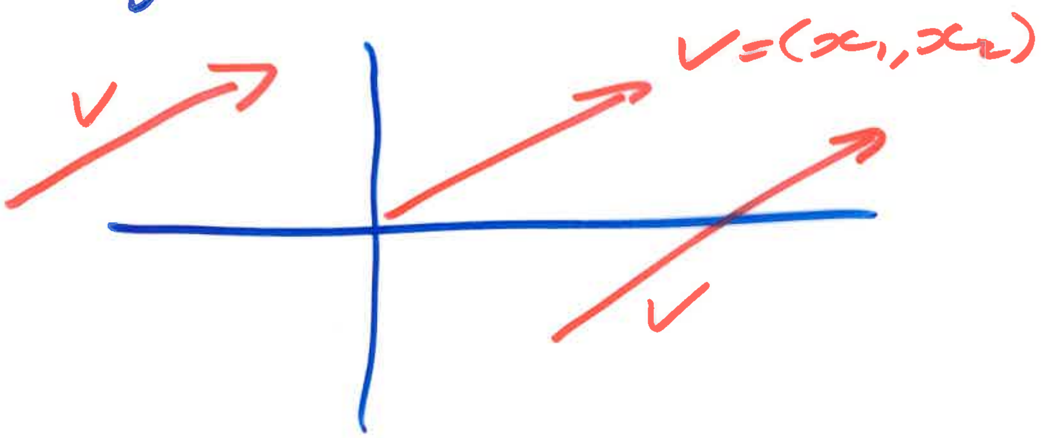
(Also called 2-dim Euclidean Space).

OR as a 2-dimensional vector $v=(x_1,x_2)$ i.e., the directed line segment from $(0,0)$ to (x_1,x_2) in the Cartesian Plane

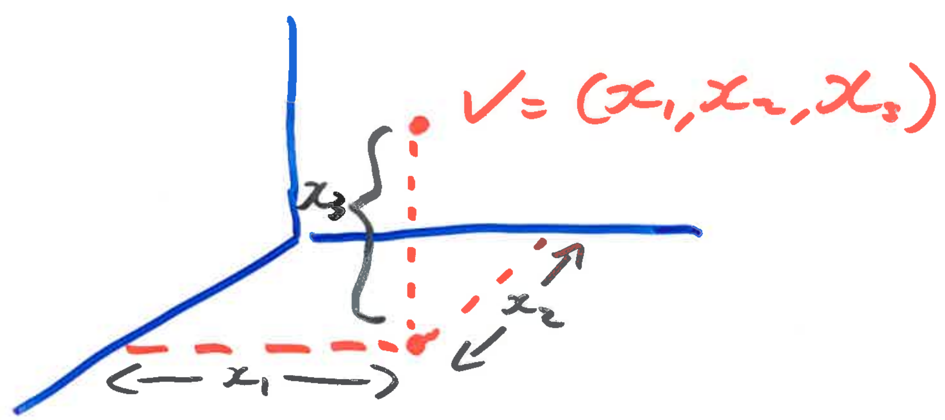


(So it is a quantity with direction and magnitude or length.)

OR: Base the vector picture at any point in the plane



$n=3$:



Any element $v = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ is called an n-dimensional vector

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Operations on vectors

(i) Scalar multiplication (or scaling).

If $v = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $r \in \mathbb{R}$.

then $rv = (rx_1, rx_2, \dots, rx_n) \in \mathbb{R}^n$ is a vector in the same direction as v (if $r > 0$) but r times as long.

(if $r < 0$, rv is in the opposite direction to v, and r times as long.).

Ex: $\left.\begin{array}{l} v = (1,2,3) \\ r = 2 \end{array}\right\} \longrightarrow 2v = (2,4,6)$

(ii) Addition of vectors

Let $v = (v_1, v_2, \dots, v_n)$, & $u = (u_1, u_2, \dots, u_n)$

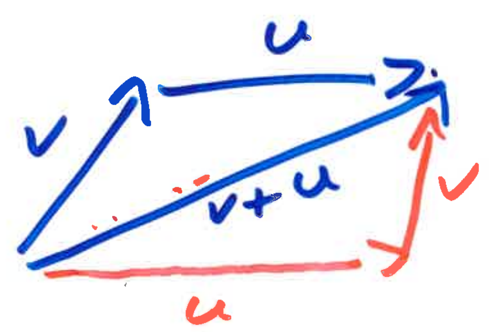
Then $u + v = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$

Note: $u+v = v+u$.

Ex: $v = (1,2) \qquad u = (2,1)$

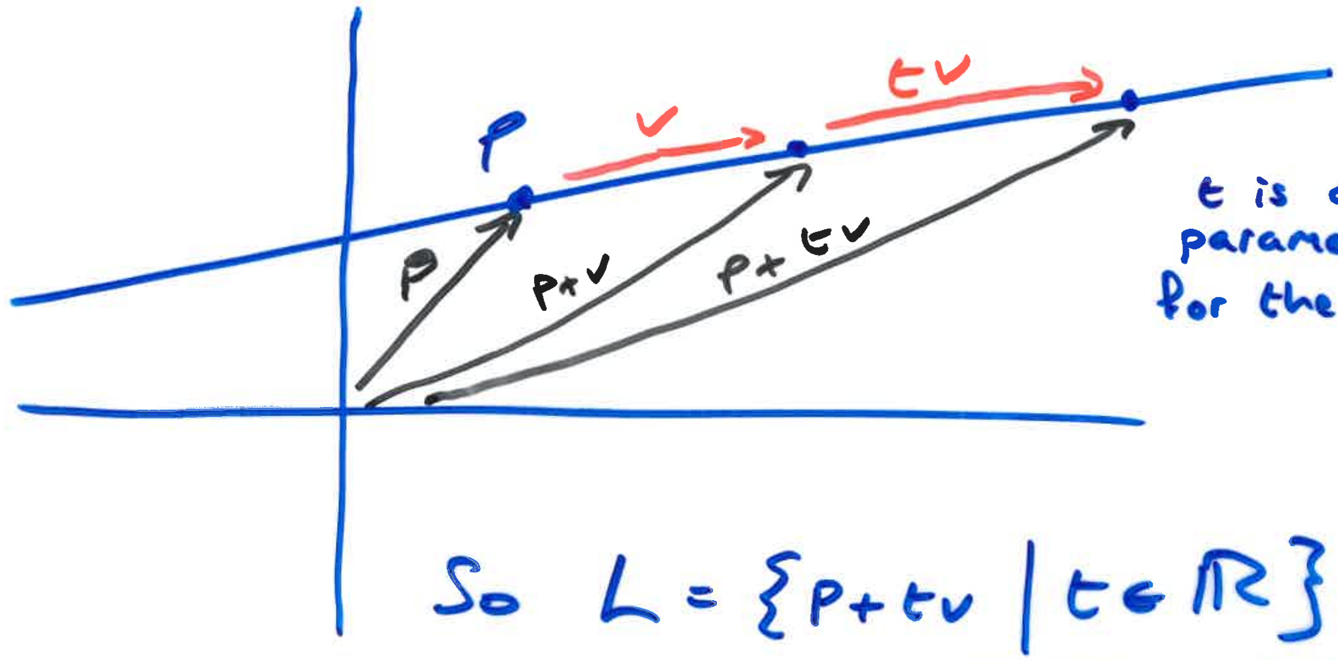
$$v+u = (1+2, 2+1) = (3,3)$$
$$u+v = (2+1, 1+2) = (3,3)$$

Geometric Picture: Place u at the tip of v and $v+u$ is the vector from the base of v to the tip of u.



Application: The parametric equation of a line in $\mathbb{R}^n$.
(we will draw it in the case $n=2$.)

To describe a line L we need to specify the direction of the line $v \neq 0$ and a point P on the line.



t is a parameter for the line.

So $L = \{p + tv \mid t \in \mathbb{R}\}$

Ex: Find the line (i.e., the parametric equation) in $\mathbb{R}^3$ in the direction $v = (1,2,3)$ passing through the point $p = (2,4,6)$.

Ans: $L = \{(2,4,6) + t(1,2,3) \mid t \in \mathbb{R}\}$

$= \{(2,4,6) + (t, 2t, 3t) \mid t \in \mathbb{R}\}$

$= \{(2+t, 4+2t, 6+3t) \mid t \in \mathbb{R}\}$.

i.e., L consists of points in $\mathbb{R}^3$ where

$x_1 = 2+t, \quad x_2 = 4+2t, \quad x_3 = 6+3t$

$\Rightarrow t = x_1 - 2 = \dfrac{x_2 - 4}{2} = \dfrac{x_3 - 6}{3}$.