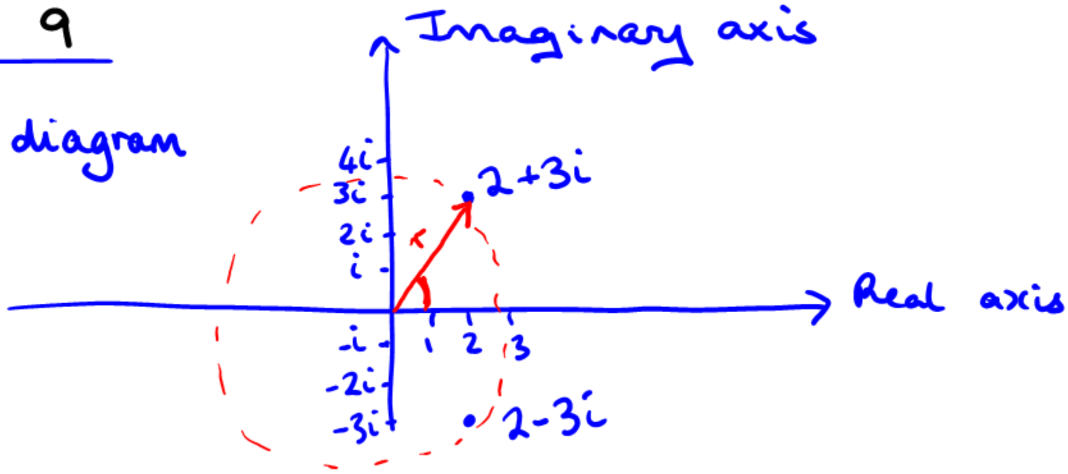
Lecture 9

Argand diagram



Reminder - division by a complex number:

Examples

(1) $\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i}\cdot\left(\dfrac{1+i}{1+i}\right) = \dfrac{(1+i)^2}{1^2-i^2} = \dfrac{(1+i)^2}{1-(-1)} = \dfrac{1^2+i^2+2i}{2}$

$= \dfrac{1-1+2i}{2} = i$

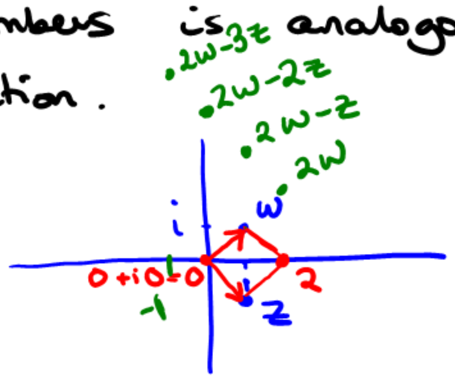
(2) $\dfrac{-3-3i}{2+2i} = \dfrac{-3-3i}{2+2i}\cdot\left(\dfrac{2-2i}{2-2i}\right) = \ldots\ldots = -3/2$

$\dfrac{-3\cancel{(1+i)}}{2\cancel{(1+i)}}$ ← shorter than pursuing the method of multiplication by $\dfrac{2-2i}{2-2i}$.

Note that on an Argand diagram addition and subtraction of complex numbers is analogous to vector addition and subtraction.

Eg. $w = 1+i$

$z = 1-i$



See that $(0+0i)$, w, z, $z+w$ form the vertices of a parallelogram, with $z+w=2$.

Algebraically,

$z+w = (1+1)+(i-i)$

$= 2$

Example Let $u = 3-4i$, $w = 1+i$, $z = 1-i$.

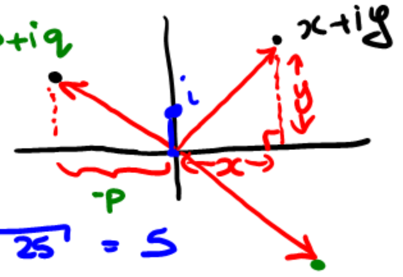
Then $u+w = 4-3i$ (move 1 right and 1 up from u)

Note $2w-3z = -1+5i$ (start at w, double it - get to $2+2i$; then $-z$ means left, 1 up; do this 3 times to get $-1+5i$.

$2+2i \xrightarrow{-z} 1+3i \xrightarrow{-z} 0+4i \xrightarrow{-z} -1+5i$

Definition For $z = x+iy$, we define the modulus $|z|$ to be

$$|z| = \sqrt{x^2+y^2}$$



Examples: for $z = 4-3i$, find $|z|$.

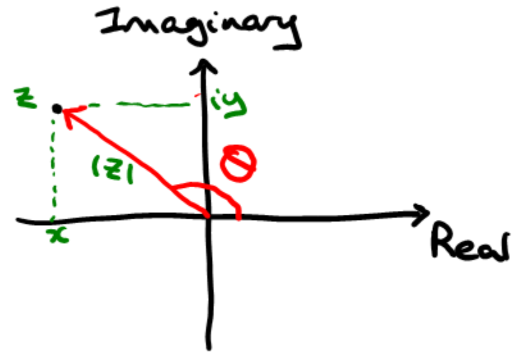
Soln: $|z| = \sqrt{4^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

For $z = \dfrac{1+i}{1-i}$, find $|z|$.

Soln: From earlier, $\dfrac{1+i}{1-i} = i$ and so $|z| = |0+1i|$

$$= \sqrt{0^2+1^2}$$
$$= \sqrt{1} = 1.$$

(Quick aside: $\sqrt{4} = 2$ as by convention the symbol '$\sqrt{\ }$' gives the positive square root.)

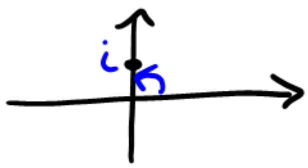


Note: if we interpret $z = x+iy$ on an Argand diagram, then $|z|$ = length of the ray from 0 to z.

z also subtends an angle θ measured anticlockwise (↺) from the positive real axis.

Definition: We call the angle θ subtended by z the argument of z, denoted $\text{Arg}(z)$.

Problem for $z = i$, determine $\text{Arg}(z)$.



$\text{Arg}(i) = \pi/2$ (or $90°$)

For $z = -3-3i$, find $\text{Arg}(z)$.