Lectures 6 and 7

Problem 8 If fallout shelters are built, other countries will feel endangered and our people will get a false sense of security. If other countries feel endangered, they may start a preventative war. If our people will get a false sense of security, they will put less effort into preserving peace. If fallout shelters are not built, we run the risk of tremendous losses in the event of war.

Hence, either other countries may start a preventative war and our people will put less effort into preserving peace, or we run the risk of tremendous losses in the event of war.

Let F denote "fallout shelters are built"

E " "other countries will feel endangered"

S " "our people will get false sense of security".

Formalising first sentence of argument: $F \Rightarrow (E \wedge S)$ ①

Let P denote "other countries may start a preventative war"

Formalising next sentence: ② $E \Rightarrow P$

Let L denote "they will put less effort in preserving peace"

Logic: ③ $S \Rightarrow L$

Let R denote "we run the risk of ..... war"

Logic: ④ $\neg F \Rightarrow R$

(Hence:) ⑤ $(P \wedge L) \vee R$

① ∧ ② ∧ ③ ∧ ④ ⇒ ⑤

ie $\Big((F \Rightarrow (E \wedge S)) \wedge (E \Rightarrow P) \wedge (S \Rightarrow L) \wedge (\neg F \Rightarrow R) \Big) \Rightarrow (P \wedge L) \vee R$

ie of the form $A \Rightarrow B$

We want to know if this implication can be false. This can only happen if A is true and B is false.

Suppose that $B = (P \wedge L) \vee R$ is false.

This means that R is false, and $P \wedge L$ is false

($P \wedge L$ is false means that one of the following options holds:

P true, L false

P false, L true

P false, L false.)

Now consider

$A: \big(F \Rightarrow (\underset{①}{E} \wedge S)\big) \wedge (\underset{②}{E} \Rightarrow P) \wedge (\underset{③}{S} \Rightarrow L) \wedge (\underset{④}{\neg F} \Rightarrow R)$.

Recall that R is false, whereas P and L have options (as before).

Assume A is true. This means that ①, ②, ③ and ④ must each be true.

④ $\neg F \Rightarrow R$ is true, and R is false yields $\neg F$ false and so F must be true.
(false, false)

Now look at ①: $F \Rightarrow (E \wedge S)$ is true, and F is true yields $E \wedge S$ is true giving E true and S true.

Next consider ②: $E \Rightarrow P$ is true and E is true, giving P true.

Finally, ③: $S \Rightarrow L$ is true, S is true and so L is true.

So we have: R false, F true, E true, S true, P true, L true

BUT $P \wedge L$ is false (from earlier work)

This is a contradiction, and so our truth function cannot ever be false ie it is a tautology, and so the corresponding argument is valid.