Lecture 5 — Solving logic problems

Problem 5 (from the list of 10!)

(i) Decide whether $(\neg P) \vee Q$ is logically equivalent to $(\neg Q) \vee P$.

(ii) Decide whether $\neg(P \Leftrightarrow Q)$ is logically equivalent to $P \Leftrightarrow (\neg Q)$.

Solution

(i)

P	Q	¬P	(¬P)∨Q	¬Q	(¬Q)∨P
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Not logically equivalent

Note that we only need one 'disagreement' to demonstrate that they are not logically equivalent.

(ii)

P	Q	¬Q	P⇔Q	¬(P⇔Q)	P⇔(¬Q)
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	F

∴ logically equivalent.

Problem 7 Represent the following argument by a truth function. Then decide if it is logically valid.

"If the temperature and air pressure remained constant, there was no rain. The temperature did remain constant. Therefore, if there was rain, then the air pressure did not remain constant."

Solution: P denotes "Temperature remained constant"

Q " " "Air pressure remained constant"

R denotes "there was rain."

$P \wedge Q \Rightarrow \neg R \,.\; P \,.\; \therefore \quad R \Rightarrow \neg Q$

Finally, the corresponding truth function is

$$S := \Big((P \wedge Q \Rightarrow \neg R) \wedge P \Big) \Rightarrow (R \Rightarrow \neg Q) \,.$$

Next build truth table:

P	Q	R	$P \wedge Q$	$\neg R$	$\neg Q$	$(R \Rightarrow \neg Q)$	$(P \wedge Q \Rightarrow \neg R)$	$(P \wedge Q \Rightarrow \neg R) \wedge P$	S
T	T	T	T	F	F	F	F	F	T
T	T	F	T	T	F	T	T	T	T
T	F	T	F	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T	T
F	T	T	F	F	F	F	T	F	T
F	T	F	F	T	F	T	T	F	T
F	F	T	F	F	T	T	T	F	T
F	F	F	F	T	T	T	T	F	T

So the truth function is a tautology, and thus the argument is valid.

Problem 10 Find a formula in terms of $\wedge, \neg, \vee$ for the truth function defined by the truth table:

P	Q	$\neg Q$	$\neg P$	$P \wedge Q$	$\neg(P \wedge Q)$	$P \Rightarrow \neg Q$	$(\neg P) \vee (\neg Q)$
T	T	F	F	T	F	F	F
T	F	T	F	F	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	F	T	T	T
					✓	✓	✓

$\neg P \vee \neg Q$

$\neg(P \wedge Q)$