Lecture 4

Answering last day's question:

P	Q	$\neg P$	$P \wedge (\neg P)$	$(P \wedge (\neg P)) \Rightarrow Q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

↗ tautology since output is always true (T).

Summary: We have defined the logical connectives $\wedge$, $\vee$, $\neg$, $\Rightarrow$.

Note that "$\Rightarrow$" can be expressed in terms of $\vee$, $\neg$ only; verify that

$$P \Rightarrow Q \equiv (\neg P) \vee Q$$

P	Q	$\neg P$	$(\neg P) \vee Q$	$P \Rightarrow Q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

↖ these two columns agree for every possible input, so

$P \Rightarrow Q \equiv (\neg P) \vee Q$.

Another useful connective is denoted by $\Leftrightarrow$.

$P \Leftrightarrow Q$ is shorthand for $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

and it can be spoken as

"P if and only if Q"

sometimes shortened to 'iff'

The truth table for $P \Leftrightarrow Q$ is (check!)

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

as promised!

Let's check this:

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

✓

We model logical arguments by truth functions.

Recall: A truth function is called a tautology if its truth value is T, no matter what the input.

Definition: An argument is logically valid if the corresponding truth function is a tautology.

Problem: Is the following logically valid?

"If Murphy is a communist, then Murphy is an atheist. Murphy is an atheist. Hence, Murphy is a communist."

Solution: Let P := "Murphy is a communist"
Q := "Murphy is an atheist"

Express argument symbolically:

If P then Q. Q. Hence, P.

More succinctly, $P \Rightarrow Q \;.\; Q \;.\; \therefore P$ (notation for 'therefore / hence')

Now set up the corresponding truth function:

If (all hypotheses) then (conclusion)

ie If $((P \Rightarrow Q) \wedge Q)$ then P

ie $((P \Rightarrow Q) \wedge Q) \Rightarrow P$ is the corresponding truth function.

Is it a tautology?

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \wedge Q$	$((P \Rightarrow Q) \wedge Q) \Rightarrow P$
T	T	T	T	T
T	F	F	F	T
→ F	T	T	T	(F) ← Not a tautology
F	F	T	F	T

So argument is not valid

(it fails if P is false and Q is true - as we expected.)