Lecture 3

* Tutorials start tomorrow, Thursday 23rd Jan:

MA160	Thurs 12noon	AC204
MA135	Thurs 1 pm	MY126

Problem: For each Boolean function below, compute its "truth table":

(i) $f(x,y) = xy \bmod 2$

(ii) $f(x,y) = x+y \bmod 2$

x	y	(i) $xy \bmod 2$	(ii) $x+y \bmod 2$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

Problem: Determine a function $f(x,y) \bmod 2$ with the following table:

x	y	$f(x,y)$
1	1	1
1	0	0
0	1	1
0	0	1

Ans: (strategy - trial and error)

$$f(x,y) = x + 1 + xy \pmod 2$$

Conditional statements:

$P \Rightarrow Q$ "P implies Q"

(or $P \rightarrow Q$) "If P then Q"

Motivating Example: from Intoxicating Liquor Act (2008):

"If (x drinks alcohol) then (x is 18 or older)"

Under what (if any) circumstances can this logical statement be false (ie break)? ie can this be unlawful?

Based on our discussion (that the only circumstance under which this law is broken is when "x drinks alcohol" is true but "x is 18 or older" is false, we define the connective $\Rightarrow$ by the following truth table:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

hypothesis

Note: The two statements "$P \Rightarrow Q$" and "$(\neg P) \vee (P \wedge Q)$" have the same truth table – check this:

P	Q	$\neg P$	$P \wedge Q$	$(\neg P) \vee (P \wedge Q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Definition: A logical expression is called a tautology if it is always true.

Example: $P \vee (\neg P)$ Check:

P	$\neg P$	$P \vee (\neg P)$
T	F	T
F	T	T

taut-ology

Definition: A logical expression is a contradiction if it is always false.

Example: $P \wedge (\neg P)$. Verify:

P	$\neg P$	$P \wedge (\neg P)$
T	F	F
F	T	F

Example: Show that $(P \wedge \neg P) \Rightarrow Q$ is a tautology.

N.B: this is a contradiction ie only ever is F.