Recall that we had

$$Av = b$$

where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}$, $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $b = \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$.

With $D = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$, we found that

$DA = I$ so that D is the inverse matrix of A ie $D = A^{-1}$.

$$\begin{aligned} Av &= b \\ \Rightarrow \quad \underbrace{A^{-1}(A}v) &= A^{-1}b \\ \Rightarrow \quad Iv &= A^{-1}b \\ \Rightarrow \quad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= A^{-1}b \end{aligned}$$

<u>Problem</u>: Solve $\left.\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 5y + 5z &= 2 \\ 3x + 8y + 6z &= 3 \end{aligned}\right\}$

<u>Solution</u>: Express as $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

ie $Av = b$

The solution is $v = A^{-1}b$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

Question: How do we find A^{-1}??

Answer 1 : Use cofactor method (Semester 1 method)

Answer 2 : Use Gauss-Jordan method

(a Gaussian Elimination or row operations)

Gauss-Jordan method for finding the inverse of an $n \times n$ matrix A

$$(A : I) \xrightarrow{\text{row operations}} (I : B)$$

3×6 matrix

Fact: $B = A^{-1}$

There are three kinds of row operation:

① $R_i \to R_i + \lambda R_j$ $(\lambda \in \mathbb{R},\ i \neq j)$

② $R_i \leftrightarrow R_j$ (switch R_i with R_j)

③ $R_i \to \lambda R_i$ $(\lambda \neq 0)$ (multiply or divide a row by a scalar)

Example: Let's compute $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}^{-1}$ with this method.

Solution: $\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{pmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{} \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -3 & 0 & 1 \end{pmatrix}$

$A : I$

$\xrightarrow[R_3 \to R_3 - 2R_2]{}$ To be continued!