Example Let $D = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}$

Compute DA where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}$

<u>Answer</u>:

$$DA = \begin{pmatrix} 10(1)-12(2)+5(3) & 10(2)-12(5)+5(8) & 10(3)-12(5)+30 \\ -3(1)+3(2)-1(3) & -3(2)+3(5)-1(8) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= I$, the identity matrix

Note: A matrix is called <u>square</u> if the number of rows is the same as the number of columns. For example, D and A are both square matrices - as is I.

Note further that $\left.\begin{array}{r} IE = E \\ \text{and} \quad EI = E \end{array}\right\}$ for any 3×3 matrix E.

If A is a matrix and $AB = I$, then we call B the inverse matrix of A and we write it as $B = A^{-1}$.

Recall our system of equations that can be written as

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

or $$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}.$$

Strategy: $$DA\begin{pmatrix} x \\ y \\ z \end{pmatrix} = D\begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$ (multiplying by D on the left (both sides))

$$\Rightarrow I\begin{pmatrix} x \\ y \\ z \end{pmatrix} = D\begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$ (since $DA = I$)

(since $Iv = v$, $IE = E$ etc)

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{pmatrix}\begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}$$

$$= \begin{pmatrix} 100 - 12(21) + 5(31) \\ -3(10) + 3(21) - 1(31) \\ -10 + 42 - 31 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

ie $x = 3$, $y = 2$, $z = 1$.