Lecture 2

Another useful connective is 'NOT'.
This is defined by the truth table:

| $P$ | NOT $P$ |
|---|---|
| T | F |
| F | T |

Notation: We write $P \vee Q$ for "P OR Q"
$P \wedge Q$ for "P AND Q"
$\neg P$ for "NOT P"
(also called the negation of P.)

You need to know the above three truth table definitions for $\vee$, $\wedge$ and $\neg$.

We can use these connectives to build complicated logical expressions such as

$$\underbrace{\overbrace{((\neg P) \wedge Q)}^{A}} \vee \underbrace{(P \wedge R)}_{B}$$

| $P$ | $Q$ | $R$ | $\neg P$ | $(\neg P) \wedge Q$ | $P \wedge R$ | $A \vee B$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T . |
| T | T | F | F | F | F | F |
| T | F | T | F | F | T | T . |
| T | F | F | F | F | F | F |
| F | T | T | T | T | F | T . |
| F | T | F | T | T | F | T . |
| F | F | T | T | F | F | F |
| F | F | F | T | F | F | F |

If spoken "P OR Q AND P",

is the intention P OR (Q AND P) ie $P \vee (Q \wedge P)$, or (1)

is it (P OR Q) AND P ie $(P \vee Q) \wedge P$ ? (2)

Does it matter?

| $P$ | $Q$ | $Q \wedge P$ | (1) $P \vee (Q \wedge P)$ | $P \vee Q$ | (2) $(P \vee Q) \wedge P$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | T | T |
| F | T | F | F | T | F |
| F | F | F | F | F | F |

The two relevant columns are identical, so we write that $P \vee (Q \wedge P) \equiv (P \vee Q) \wedge P$ and we say that $P \vee (Q \wedge P)$ is <u>logically equivalent</u> to $(P \vee Q) \wedge P$. So it doesn't matter in this case: one could write $P \vee Q \wedge P$ without brackets if one wanted.

<u>Problem</u>: Write out the truth table for $(\neg P) \vee (P \wedge Q)$.

| $P$ | $Q$ | $\neg P$ | $P \wedge Q$ | $(\neg P) \vee (P \wedge Q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | F | T | F | T |