Linear equations through matrices

The linear system
$$\begin{cases} 1x + 2y + 3z = 10 \\ 2x + 5y + 5z = 21 \\ 3x + 8y + 6z = 31 \end{cases}$$
can be succintly expressed as
$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{v} = \underbrace{\begin{pmatrix} 10 \\ 21 \\ 31 \end{pmatrix}}_{b}$$

$$\boxed{Av = b}$$

A is a 3×3 matrix (3 rows, 3 columns)

v is a 3×1 matrix

b " " 3×1 matrix

An $m \times n$ matrix is an array of numbers with m rows and n columns.

Some terminology:

An $m \times 1$ matrix is a column vector.

A $1 \times n$ matrix is a row vector.

For example,

$\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$ is a column vector;

$\begin{pmatrix} 0 & -3 & 14 \end{pmatrix}$ is a row vector, as is

$\begin{pmatrix} 1 & 2 & 4 & 7 & 11 & 3 & -2 \end{pmatrix}$.

How do we multiply matrices?

A row vector R of length n, and a column vector C of the same length (n) can be multiplied as follows:

Suppose $R = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}$, $C = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$.

$1 \times n$ $\qquad$ $n \times 1$

Then $RC = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$.

1×1 matrix

Eg $\begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ -4 \end{pmatrix} = 3(1) + 5(-4) = -17$

$$\begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} -17 & 3(2)+5(1) \end{pmatrix} = \begin{pmatrix} -17 & 11 \end{pmatrix}$$

1×2 $\qquad$ 2×2 $\qquad$ 1×2

Two 3×3 matrices can be multiplied as follows:

Suppose $A = \begin{pmatrix} - R_1 - \\ - R_2 - \\ - R_3 - \end{pmatrix}$ and $B = \begin{pmatrix} | & | & | \\ C_1 & C_2 & C_3 \\ | & | & | \end{pmatrix}$.

3×3 $\qquad$ 3×3

Then $AB = \begin{pmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \\ R_3C_1 & R_3C_2 & R_3C_3 \end{pmatrix}$.

3×3