Lecture 17

Recall : A linear equation of the form

$$ax + by + cz = d$$

determines a plane in $\mathbb{R}^3$.

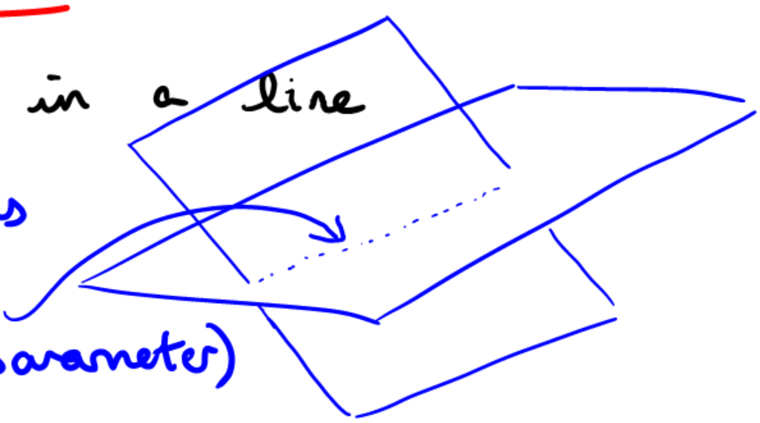
Observation : A system of two linear equations

$$\left.\begin{aligned} ax + by + cz &= d \\ a'x + b'y + c'z &= d' \end{aligned}\right\} (*)$$

describes two planes in $\mathbb{R}^3$.

For such a system, either

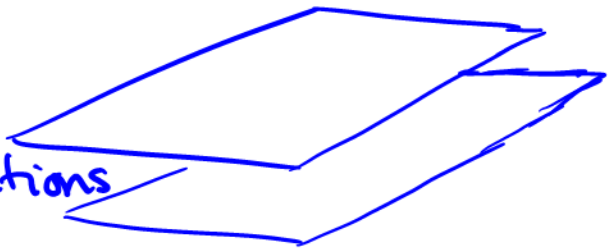
- two planes intersect in a line



$\Leftrightarrow$ the system $(*)$ has infinitely many solutions (determined by one parameter)

or

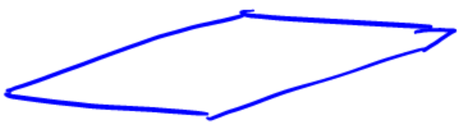
- the two planes are parallel (never meet)



$\Leftrightarrow$ the system has no solutions

or

- the planes are identical



$\Leftrightarrow$ the system has infinitely many solutions (determined by two parameters)

__Example__ Solve the system of equations

$$\left.\begin{aligned} 3x + 5y + 7z &= 15 \\ x + y + z &= 1 \end{aligned}\right\}.$$

__Solution__ :

$$\left.\begin{aligned} 1.x + y + z &= 1 \\ 3x + 5y + 7z &= 15 \end{aligned}\right\}\begin{matrix}(R_1) \\ (R_2)\end{matrix}$$ Use 1 as pivot.

$R_1 \leftrightarrow R_2$ (Switch rows)

$$\left.\begin{aligned} R_1: \quad x + y + z &= 1 \\ R_1 - 3R_2 \to R_2: \quad 2y + 4z &= 12 \end{aligned}\right\}$$

$$\left.\begin{aligned} R_1: \quad x + y + z &= 1 \\ \tfrac{1}{2}R_2 \to R_2: \quad y + 2z &= 6 \end{aligned}\right\}$$

Let $z = t$; then $y + 2t = 6 \Rightarrow y = 6 - 2t$

parameter

and $x + (6 - 2t) + t = 1$

giving $x = -5 + t$

Any solution $Q = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ to the system has the form

$$\begin{pmatrix} -5 + t \\ 6 - 2t \\ t \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\underset{\text{Point in } \mathbb{R}^3}{\text{Point}} + t\, \underset{\text{direction vector}}{v}$$

ie the system has as solutions all points on the line determined by $P = \begin{pmatrix} -5 \\ 6 \\ 0 \end{pmatrix}$ and the direction vector $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.