Lecture 15

TOPIC 3 : SYSTEMS OF LINEAR EQUATIONS

Example A factory requires energy, steel and labour to manufacture 3 machines A, B, C.

Resource	A	B	C	Weekly amount available
Energy	2 Mwh	3 Mwh	2 Mwh	100 Mwh
Steel	1 tonne	1 tonne	4 tonnes	70 tonnes
Labour	20 hrs	10 hrs	10 hrs	500 hrs

What production figures ensure all resources are used up?

Solution : Let's suppose we make x units of machine A,
y units of machine B,
z units of machine C.

If all resources are used up, then

$$\left.\begin{array}{lr} (1) & 2x + 3y + 2z = 100 \\ (2) & x + y + 4z = 70 \\ (3) & 20x + 10y + 10z = 500 \end{array}\right\} \text{system of linear equations}$$

$$\left.\begin{array}{lr} (1) & 2x + 3y + 2z = 100 \\ (2) \to (2) - \tfrac{1}{2}(1): & 0 - \tfrac{1}{2}y + 3z = 20 \\ (3) \to (3) - 10(1): & 0 - 20y - 10z = -500 \end{array}\right\} \text{another system of linear equations}$$

$$\left\{\begin{array}{l} \tfrac{1}{2}(1): \ x + \tfrac{3}{2}y + z = 50 \\ (2) \ \ x + y + 4z = 70 \end{array}\right.$$

Note well: It has the SAME solution set (ie values of x, y and z) as the original system but it has the advantage of being simpler.

$$\begin{aligned}
&① \quad & 2x + 3y + 2z &= 100 \\
&② \quad & -\tfrac{1}{2}y + 3z &= 20 \\
&③ \to ③ - 40②: \quad & 0 - 130z &= -1300
\end{aligned} \Bigg\} \text{ Simpler again}$$

$$\overbrace{(-20y + 20y)}^{0}$$

From the (new) last equation, we have

$$-130z = -1300 \Rightarrow z = \frac{130\cancel{0}}{13\cancel{0}} = 10$$

Back substitute into ②: $-\tfrac{1}{2}y + 3(10) = 20$

$$\Rightarrow -\tfrac{1}{2}y = -10$$

$$\Rightarrow y = 20$$

Finally, back substitute into ①:

$$2x + 3(20) + 2(10) = 100$$

$$\Rightarrow 2x = 100 - 80 = 20$$

$$\Rightarrow x = 10$$

Solution set: $x = 10$, $y = 20$, $z = 10$.

The above process is known as Gaussian Elimination.